

$$= \max_{\underline{v}_c} \frac{\| E_{IC} \underline{v}_c + K_I^{-1} K_{IC} \underline{v}_c \|^2_{K_I}}{\| \underline{v}_c \|^2_{S_c}}$$

$$= \max_{\underline{v}_c} \frac{\| \begin{bmatrix} \underline{v}_c \\ E_{IC} \underline{v}_c \end{bmatrix} - \begin{bmatrix} \underline{v}_c \\ -K_I^{-1} K_{IC} \underline{v}_c \end{bmatrix} \|_K^2}{\| \underline{v}_c \|^2_{S_c}}$$

Since

$$\begin{aligned} & \left(\begin{bmatrix} \underline{v}_c \\ E_{IC} \underline{v}_c \end{bmatrix}, \begin{bmatrix} \underline{v}_c \\ -K_I^{-1} K_{IC} \underline{v}_c \end{bmatrix} \right)_K = \\ & = \left(\underbrace{\begin{pmatrix} K_c & K_{CI} \\ K_{IC} & K_I \end{pmatrix}}_{S_c} \begin{bmatrix} \underline{v}_c \\ -K_I^{-1} K_{IC} \underline{v}_c \end{bmatrix}, \begin{bmatrix} \underline{v}_c \\ E_{IC} \underline{v}_c \end{bmatrix} \right) \\ & = \begin{bmatrix} (K_c - K_{CI} K_I^{-1} K_{IC}) \underline{v}_c \\ (K_{IC} - K_I K_I^{-1} K_{IC}) \underline{v}_c \end{bmatrix} = \begin{bmatrix} S_c \underline{v}_c \\ \mathbf{0} \end{bmatrix} \\ & = (S_c \underline{v}_c, \underline{v}_c) = \| \underline{v}_c \|^2_{S_c} \end{aligned}$$

and

$$\| \begin{bmatrix} \underline{v}_c \\ -K_I^{-1} K_{IC} \underline{v}_c \end{bmatrix} \|_K^2 = (S_c \underline{v}_c, \underline{v}_c) = \| \underline{v}_c \|^2_{S_c},$$

we get together with assumption (P1)

$$\begin{aligned} & \left\| \begin{bmatrix} \underline{v}_c \\ E_{IC} \underline{v}_c \end{bmatrix} - \begin{bmatrix} \underline{v}_c \\ -K_I^{-1} K_{IC} \underline{v}_c \end{bmatrix} \right\|_K^2 = \\ & = \left\| \begin{bmatrix} \underline{v}_c \\ E_{IC} \underline{v}_c \end{bmatrix} \right\|_K^2 - 2 \left(\begin{bmatrix} \underline{v}_c \\ E_{IC} \underline{v}_c \end{bmatrix}, \begin{bmatrix} \underline{v}_c \\ -K_I^{-1} K_{IC} \underline{v}_c \end{bmatrix} \right)_K + \left\| \begin{bmatrix} \underline{v}_c \\ -K_I^{-1} K_{IC} \underline{v}_c \end{bmatrix} \right\|_K^2 \\ & = \left\| \begin{bmatrix} \underline{v}_c \\ E_{IC} \underline{v}_c \end{bmatrix} \right\|_K^2 - 2 \| \underline{v}_c \|^2_{S_c} + \| \underline{v}_c \|^2_{S_c} \leq \\ & \stackrel{(P1)}{\leq} C_E^2 \| \underline{v}_c \|^2_{S_c} - \| \underline{v}_c \|^2_{S_c} = (C_E^2 - 1) \| \underline{v}_c \|^2_{S_c} \text{ q.e.d.} \end{aligned}$$