- <u>Lemma 5.4.</u> (μ-estimate)

  Let us assume that there exists a positive constant $c_E = \text{const} > 0$ ($c_E \geq 1$ due to Ex. 5.2) such that inequality

  (81) $\left\| \begin{bmatrix} \underline{v}_C \\ E_{IC}\underline{v}_C \end{bmatrix} \right\|_K \leq c_E \|\underline{v}_C\|_{S_C} \quad \forall \underline{v}_C \in \mathbb{R}^{N_C}$

  holds, where $E_{IC} := -B_I^{-1} K_{IC} : \mathbb{R}^{N_C} \to \mathbb{R}^{N_I}$

  and $\|\cdot\|_K^2 = (K\cdot,\cdot)$, $\|\cdot\|_{S_C}^2 := (S_C\cdot,\cdot)$.

  Then the (sharp) estimate

  (82) $\mu = \varrho(S_C^{-1}T_C) \leq c_E^2 - 1$

  is valid.

<u>Proof</u> Using the identity

$$\left\| \begin{bmatrix} \underline{v}_C \\ E_{IC}\underline{v}_C \end{bmatrix} - \begin{bmatrix} \underline{v}_C \\ -K_I^{-1}K_{IC}\underline{v}_C \end{bmatrix} \right\|_K^2 =$$

$$= \left(\begin{bmatrix} K_C & K_{CI} \\ K_{IC} & K_I \end{bmatrix} \left[\begin{bmatrix} \underline{v}_C \\ E_{IC}\underline{v}_C \end{bmatrix} - \begin{bmatrix} \underline{v}_C \\ -K_I^{-1}K_{IC}\underline{v}_C \end{bmatrix} \right], \left[\quad \right] \right)$$

$$= \| E_{IC}\underline{v}_C + K_I^{-1}K_{IC}\underline{v}_C \|_{K_I}^2 \quad \forall \underline{v}_C \in \mathbb{R}^{N_C},$$

we get the following representation of μ:

$$\mu = \max_{\underline{v}_C} \frac{(T_C\underline{v}_C, \underline{v}_C)}{(S_C\underline{v}_C, \underline{v}_C)}$$

$$= \max_{\underline{v}_C} \frac{(K_{CI}(K_I^{-1} - B_I^{-T})K_I(K_I^{-1} - B_I^{-1})K_{IC}\underline{v}_C, \underline{v}_C)}{(S_C\underline{v}_C, \underline{v}_C)}$$

$$= \max_{\underline{v}_C} \frac{\|K_I^{-1}K_{IC}\underline{v}_C - B_I^{-1}K_{IC}\underline{v}_C\|_{K_I}^2}{\|\underline{v}_C\|_{S_C}^2} =$$