- <u>Preconditioner:</u> $C = C^T > 0$ SPD

1. $\kappa(C^{-1}K) << \kappa(K)$

   asym. optimal: $\kappa(C^{-1}K) = O(1)$ for $h \to 0$, $J \to \infty$

   Spectral equivalence inequalities:

   $\underline{\gamma}\, C \le K \le \overline{\gamma}\, C \Rightarrow \kappa(C^{-1}K) \le \overline{\gamma}/\underline{\gamma}$

   $= O(\overline{\gamma}/\underline{\gamma})$

   $K\underline{u} = \lambda C\underline{u}$: $\underline{\gamma} \underset{\approx}{\le} \lambda_{min} \le \lambda(C^{-1}K) \le \lambda_{max} \underset{\approx}{\le} \overline{\gamma}$

   eigenvalues

2. $\underline{\hat{w}} = C^{-1}\underline{\hat{d}} \Rightarrow$ <u>fast</u>, i.e. $O(h^{-d}(\ln h^{-1})^r)$ ops

3. Communication in 2.) $\underline{\hat{w}} = C^{-1}\underline{\hat{d}}$:

   a) Typ I $= \sum_j A_{C_j}^T$ Typ II, i.e. only 1 NN-Communication!

   b) + "Cross-point-Communication (CP-Com.)

   c) + Coarse Grid Solve

- <u>First Candidate</u> = <u>Simple DD preconditioner (65):</u>

Coarse Grid Solver

CP-Communication

NN-Communication

$\underline{\hat{w}} = C^{-1}\underline{\hat{d}}$	$\hat{w}_C = C_C^{-1} \sum_{j=1}^{J} A_{C_j}^T (\underline{\hat{d}}_{C_j} - K_{C_j I_j} C_{I_j}^{-1} \underline{\hat{d}}_{I_j})$ $\hat{w}_{I_j} = C_{I_j}^{-1}(\underline{\hat{d}}_{I_j} - K_{I_j C_j}\hat{w}_{C_j})$, $\hat{w}_{C_j} = A_{C_j} w_C$ $j = \overline{1, J}$

- Reference: C.C. Douglas, G. Haase, U. Langer, SIAM, 2003