

Preconditioner: $C = C^T > 0$ SPD

1. $\kappa(C^{-1}K) \ll \kappa(K)$

asym. optimal: $\kappa(C^{-1}K) = O(1)$ for $h \rightarrow 0$,

Spectral equivalence inequalities: $J \rightarrow \infty$

$$\underline{\lambda} G \leq K \leq \bar{\lambda} G \quad \rightarrow \quad \kappa(C^{-1}K) \leq \bar{\lambda} / \underline{\lambda} \approx O(\bar{\lambda} / \underline{\lambda})$$

$$K \underline{u} = \lambda C \underline{u} : \quad \underline{\lambda} \leq \lambda_{\min} \leq \lambda(C^{-1}K) \leq \lambda_{\max} \leq \bar{\lambda}$$

eigenvalues

2. $\hat{w} = G^{-1} \hat{d}$ \rightarrow fast, i.e. $O(h^{-d} (\ln h^{-1})^r)$ ops

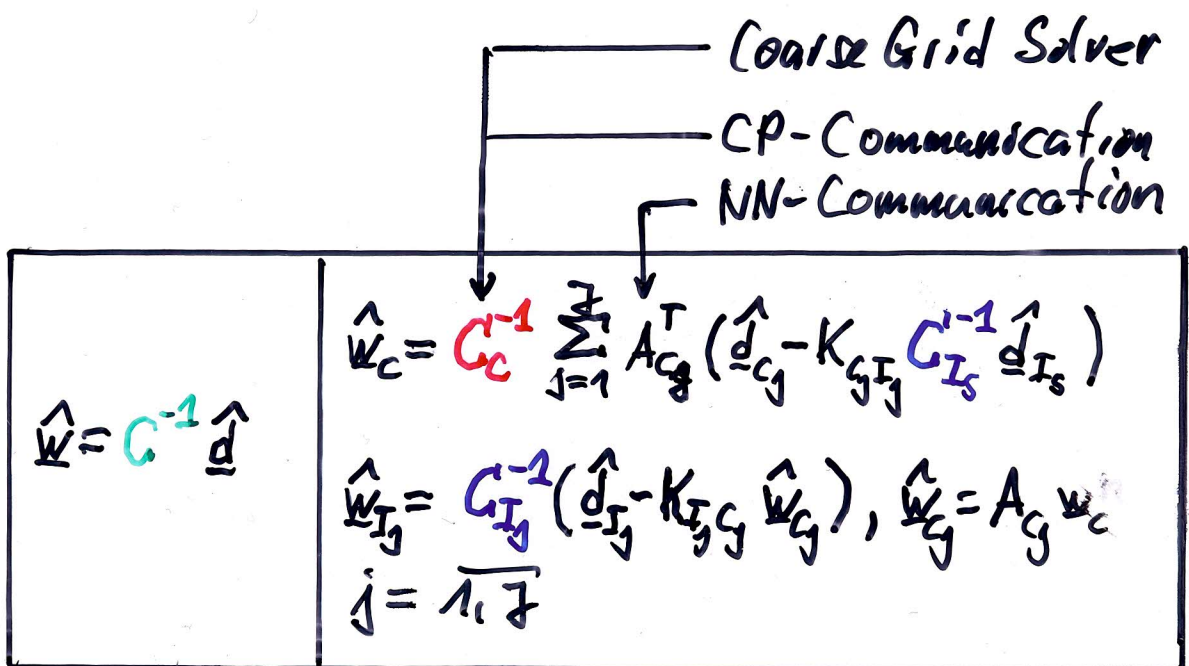
3. Communication in 2.) $\hat{w} = C^{-1} \hat{d}$:

a) Typ I $\approx \sum_i A_{C_j}^T$ Typ II, i.e. only 1 NN-Communication!

b) + "Cross-point-Communication (CP-Com.)"

c) + Coarse Grid Solve

First Candidate = Simple DD preconditioner (GS):



Reference: C.C. Douglas, G. Haase, U. Langer, SIAM, 2003