Convergence: $C = I$

$$\|\underline{u}-\underline{u}^n\|_K \le q_n \|\underline{u}-\underline{u}^0\|_K$$

with

$$\|\cdot\|_K^2 = (\cdot,\cdot)_K := (K\cdot,\cdot)$$

$$q_n = \frac{2\varrho^n}{1+\varrho^{2n}}$$

$$\varrho = \frac{1-\sqrt{\xi}}{1+\sqrt{\xi}}, \quad \xi = \frac{\underline{\gamma}}{\overline{\gamma}} \le \frac{1}{\varkappa(K)}$$

$$\underline{\gamma}\, I \le K \le \overline{\gamma}\, I$$

$$\underline{\gamma}\,(\underline{u},\underline{u}) \le (K\underline{u},\underline{u}) \le \overline{\gamma}\,(\underline{u},\underline{u}) \quad \forall \underline{u} \in \mathbb{R}^N$$

$$\underline{\gamma} \le \lambda_{min}(K) \le \lambda(I^{-1}K) \le \lambda_{max}(K) \le \overline{\gamma}$$

$$K\underline{u} = \lambda I \underline{u}$$

$\Rightarrow I(\varepsilon)$ = number of iteration $(q_{I(\varepsilon)} \le \varepsilon)$

$$= O(\sqrt{\varkappa(I^{-1}K)}\, \ln \varepsilon^{-1})$$

$$= O(\xi^{-0.5} \ln \varepsilon^{-1}) \underset{\text{PDE of 2nd order}}{=} O(h^{-1} \ln \varepsilon^{-1})$$

with $\varepsilon \in (0,1)$ - relative accuracy

Communication ($C = I$):

1. $\Sigma!$ = Scalar Product Communication (SP-Communication)

2. Type I = $\Sigma! \, A_{C_j}^T$ Type II
   → nearst neighbour communication (NN-Communication)