(P) CG		DD Par (P) CG
Serial		<u>FOR</u> <u>ALL</u> $j = 1, J$ <u>DO</u> <u>IN</u> <u>PARALLEL</u>
$\underline{u} = \underline{0}$ $\underline{x} = K\underline{u}$ $\underline{d} = \underline{f} - \underline{x}$	0. Initial step	Define $\underline{u}_j$, e.g. $\underline{u}_j = \underline{0}$, or otherwise $\underline{x}_j = K_j \underline{u}_j$ $\underline{d}_j = \underline{f}_j - \underline{x}_j$
$\underline{w} = C^{-1} \underline{d}$		$\underline{w}_C = \Sigma A^T_{C_j} \underline{d}_{C_j}$ $\underline{w}_{I_j} = \underline{d}_{I_j}$ $= C^{-1} \begin{pmatrix} \underline{d}_{C_j} \\ \underline{d}_{I_j} \end{pmatrix}$
$\underline{s} = \underline{w}$ $\beta = \underline{w}^T \underline{d}$ $\beta 0 = \beta$		$\underline{s}_j = \underline{w}_j$ $\beta_j = \underline{w}_j^T \underline{d}_j$ $\beta 1 = \Sigma \beta_j$; $\beta 0 := \beta 1$
$\underline{x} = K \underline{s}$ $\alpha = \dfrac{\underline{w}^T \underline{d}}{\underline{x}^T \underline{s}}$	Iteration 1.	$\underline{x}_j = K_j \underline{s}_j$ $\alpha_j = \underline{x}_j^T \underline{s}_j$ $\alpha 1 = \Sigma \alpha_j$ $\alpha = \beta 1 / \alpha 1$
$\hat{\underline{u}} = \underline{u} + \alpha \underline{s}$ $\hat{\underline{d}} = \underline{d} - \alpha \underline{x}$	2.	$\hat{\underline{u}}_j = \underline{u}_j + \alpha \underline{s}_j$ $\hat{\underline{d}}_j = \underline{d}_j - \alpha \underline{x}_j$
$\hat{\underline{w}} = C^{-1} \hat{\underline{d}}$	3.	$\hat{\underline{w}}_C = \Sigma A^T_{C_j} \hat{\underline{d}}_{C_j}$ $\hat{\underline{w}}_{I_j} = \hat{\underline{d}}_{I_j}$ $= C^{-1} \begin{pmatrix} \underline{d}_{C_j} \\ \underline{d}_{I_j} \end{pmatrix}$
$\beta = \dfrac{\hat{\underline{w}}^T \hat{\underline{d}}}{\underline{w}^T \underline{d}}$ $\hat{\underline{s}} = \hat{\underline{w}} + \beta \underline{s}$ $(\hat{\underline{w}}, \hat{\underline{d}}) \le \varepsilon^2 * \beta 0$? NO → 1. YES → STOP	4.	$\beta_j = \hat{\underline{w}}_j^T \hat{\underline{d}}_j$ $\beta 2 = \Sigma \beta_j$ $\beta = \beta 2 / \beta 1$; $\beta 1 := \beta 2$ $\hat{\underline{s}}_j = \hat{\underline{w}}_j + \beta \underline{s}_j$ $\beta 2 \le \varepsilon^2 * \beta 0$? NO → 1. YES → STOP

where $\hat{\underline{u}}, \hat{\underline{d}}, \ldots$ ~ new iterates, Σ = Communication + Summation