Exercise 4.1:

Show that No 1 satisfies the relation $\lambda_{min}(P_{ASM}) = \lambda_{min}(C^{-1}K) = 1$ and $\lambda_{max}(P_{ASM}) = \lambda_{max}(C^{-1}K) = L$, i.e. $\kappa(P_{ASM}) = \kappa(C^{-1}K) \simeq L$!

Remark 4.6:

1. No 1 (exact) $\longmapsto$ No 2 (inexact)

$a_\ell(\cdot,\cdot) := a(\cdot,\cdot)|_{V_\ell}$ $\qquad$ $a_\ell(\cdot,\cdot) = 2^{2(\ell-1)}(\cdot,\cdot)_{L_2}$

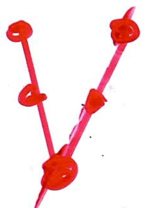
$\kappa = L$ $\qquad$ $\kappa = O(1)$

cf. Theorem 3.16 vs. 3.17 !

2. For No 3-4 ($\dim V_{\ell,j} = 1$), we have

$$\tilde{P}u = \sum_{\ell=0}^{L} \sum_{j \in \omega_\ell} \underbrace{\Phi V_{\ell,j}}_{=\varphi_{\ell,j}} \frac{a(u, \Phi V_{\ell,j})}{a_{\ell,j}(\Phi V_{\ell,j}, \Phi V_{\ell,j})} = \sum_{\ell=0}^{L} \sum_{j \in \omega_\ell} \frac{a(u, \varphi_{\ell,j})}{a_{\ell,j}(\varphi_{\ell,j}, \varphi_{\ell,j})} \varphi_{\ell,j}$$

$$\underline{\tilde{P}u} = C^{-1}K\underline{u} = \sum_{\ell=0}^{L} \sum_{j \in \omega_\ell} \frac{1}{a_{\ell,j}(\varphi_{\ell,j}, \varphi_{\ell,j})} V_{\ell,j} V_{\ell,j}^T \underbrace{K\underline{u}}$$



$$\underline{\tilde{P}(u-u^n)} = C^{-1}K(u-u^n) = C^{-1}\underline{d}^n \qquad \underline{d}^n$$

(V-cycle)

4. In Oswald's monograph (1994, B.G. Teubner, Stuttgart), one can find a very good overview over multilevel preconditioners and their analysis.
5. Griebel (1994, B.G. Teubner, Stuttgart) developed an algebraic approach to multilevel preconditioners via "generating system" $= \{\varphi_{\ell,j} : j \in \omega_\ell, \ell = \overline{0,L}\}$!
6. see also www.numa.uni-linz.ac.at/Teaching/LVA/2010w/* 2008w/*

$* =$ SemNum