Now, $\sum_{j=1}^{J}$ (50) gives

$$a(P_{ASM}u,u) = a\Big(\sum_{j=1}^{J} P_j u, u\Big) \overset{(50)}{\leq}$$

$$\leq \hat{\omega}^2 \sum_{j=1}^{J} [\Gamma \underline{c}]_j^2$$

$$\leq \hat{\omega}^2 \, \varrho(\Gamma)^2 \, \|\underline{c}\|_{\ell_2}^2$$

$$= \hat{\omega}^2 \, \varrho(\Gamma)^2 \sum_{j=1}^{J} a(E_{j-1}^* P_j E_{j-1} u, u)$$

Together with $a(P_{ASM}u,u) \geq c_L^{-2} a(u,u)$, we get

$$c_L^{-2} \hat{\omega}^{-2} \varrho(\Gamma)^{-2} a(u,u) \leq \sum_{j=1}^{J} a(E_{j-1}^* P_j E_{j-1} u, u)$$

Combining this estimate with (48) gives

$$I - E_J^* E_J^{\cancel{*}} \geq \frac{2-\omega}{c_L^2 \hat{\omega}^2 \varrho(\Gamma)^2} I \quad , \text{ i.e}$$

$$\|E_J\|_a^2 \leq 1 - \frac{2-\omega}{c_L^2 \hat{\omega}^2 \varrho(\Gamma)^2} = q_{MSM}^2$$

q.e.d.