- The definition of E_J yields

$$I = \underbrace{\underbrace{\underbrace{E_{J-1} + P_{J-1}E_{J-2}}_{(I-P_{J-1})E_{J-2}+P_{J-1}E_{J-2}} + P_{J-2}E_{J-3}}_{=E_{J-2}} + \ldots}_{=E_{J-3}} + P_1E_0 = E_0 = I,$$

that means

$$(49)\quad I = E_{J-1} + \sum_{k=1}^{J-1} P_k E_{k-1}.$$

Therefore, for all $j=\overline{1,J}$, we have the identity

$$a(P_J u,u) \overset{(43)}{=} a(P_J u, E_{J-1}u) + \sum_{k=1}^{J-1} a(P_J u, P_k E_{k-1} u) \quad (44)$$

that can be bounded by means of Lemma 3.22 as follows:

$$a(P_J u,u) \underset{(44)}{\overset{(43)}{\leq}} a(P_J u,u)^{1/2}\, 1\cdot\Big[\underset{\gamma_{JJ}}{1}\cdot a(P_J E_{J-1}u, E_{J-1}u)^{1/2} +$$

$$+ \omega \sum_{k=1}^{J-1} \gamma_{Jk}\, a(P_k E_{k-1}u, E_{k-1}u)^{1/2}\Big]$$

$$\leq a(P_J u,u)^{1/2}\, \hat{\omega}\, [\Gamma \underline{c}]_J,$$

with $\underline{c} = [c_k]_{k=\overline{1,J}}$, $c_k = a(P_k E_{k-1}u, E_{k-1}u)^{1/2}$,

and $\hat{\omega} = \max\{1,\omega\}$. Hence, we have

$$\sum_{J=1}^{J} \quad (50)\quad a(P_J u,u) \leq \hat{\omega}^2 [\Gamma\underline{c}]_J^2$$

$$a(\underbrace{\sum_{J=1}^{J} \hat{P}_J}_{\hat{P}_{ASM}} u,u) \leq \hat{\omega}^2 \sum_{J=1}^{J} - \!\!-\!\!-$$