$\sum_{j=1}^{J}$ (46) immediately yields

$$(47) \quad I - E_J^* E_J = E_0^* E_0 - E_J^* E_J = \sum_{j=1}^{J} E_{j-1}^* Q_j E_{j-1}$$

For

$$\omega = \max_{j=1,J} \sup_{w \in V} \frac{a(P_j w, P_j w)}{a(P_j w, w)} \overset{\text{mus}}{=} \max_{j=1,J} \|P_j\|_a < 2,$$

we have (cf. also Assumption 3.20)

$$\begin{aligned} a(Q_j v, v) &= a((2I - P_j) P_j v, v) = \\ &= 2a(P_j v, v) - \frac{a(P_j v, P_j v)}{a(P_j v, v)} a(P_j v, v) \\ &\geq \left[2 - \sup_{w \in V} \frac{a(P_j w, P_j w)}{a(P_j w, w)}\right] a(P_j v, v) \\ &\geq [2 - \omega]\, a(P_j v, v), \text{ i.e} \end{aligned}$$

$$Q_j \geq (2 - \omega) P_j \quad \text{wrt } a(\cdot, \cdot)$$

$P_j = P_j^* \geq 0$ wrt $a(\cdot,\cdot)$

Therefore, $Q_j \geq (2-\omega) P_j$ and (47) yields

$$(48) \quad \boxed{I - E_J^* E_J \geq \underbrace{(2-\omega) \sum_{j=1}^{J} E_{j-1}^* P_j E_{j-1}}_{\geq \alpha I \ !,\ \alpha \in (0,1) \ !}}$$

Road map:

$$\Rightarrow (1-\alpha) I \geq E_J^* E_J$$

$$\Rightarrow (1-\alpha) \underbrace{a(v,v)}_{= \|v\|_a^2} \geq a(E_J^* E_J v, v) = a(E_J v, E_J v) = \|E_J v\|_a^2$$

$$1 > q_{MSM}^2 = 1 - \alpha \geq \sup_{v \in V} \frac{\|E_J v\|_a^2}{\|v\|_a^2} = \|E_J\|_a^2 .$$