Algorithm 3.2: Inexact MSM for solving $K\underline{u} = \underline{f}$
(function ↔ vector/matrix versions)

$u^0 = \Phi \underline{u}^0 \in V$ given initial guess;

FOR $n = 0$ STEP 1 UNTIL Convergence DO

  FOR $j = 1$ STEP 1 UNTIL J DO

$$w_j^{n+(j/J)} = \Phi V_j \underline{w}_j^{n+(j/J)} \in V_j : \; a_j(w_j^{n+(j/J)}, v_j) = \langle F, v_j \rangle - a(u^{n+[(j-1)/J]}, v_j) \quad \forall v_j \in V_j$$

$$\Psi_j = \Phi V_j \quad (\updownarrow)$$

$$\underline{w}_j^{n+(j/J)} \in \mathbb{R}^{N_j} : \; C_j \underline{w}_j^{n+(j/J)} = V_j^T \underline{d}^{n+[(j-1)/J]}$$

$$u^{n+(j/J)} = u^{n+[(j-1)/J]} + w_j^{n+(j/J)}$$

$$\Phi \quad (\updownarrow)$$

$$\underline{u}^{n+(j/J)} = \underline{u}^{n+[(j-1)/J]} + V_j \underline{w}_j^{n+(j/J)}$$

  END FOR

END FOR

where $w_j^{n+(j/J)} = \tilde{P}_j (u - u^{n+[(j-1)/J]})$,

$$\underline{d}^{n+[(j-1)/J]} = \underline{f} - K \underline{u}^{n+[(j-1)/J]} .$$

Exact MSM: $a_j(\cdot,\cdot) := a(\cdot,\cdot)$,

$$C_j := V_j^T K V_j .$$