Algorithm 3.1: Exact / Inexact ASM

for solving $K\underline{u} = \underline{f}$ (function $\leftrightarrow$ vector / matrix)

$u^0 = \Phi\, \underline{u}^0 \in V$ given initial guess;
τ given iteration parameter (Richardso $\rightarrow$ CG)
FOR $n = 0$ STEP 1 UNTIL Convergence DO
  FOR ALL $j \in \{1, 2, \dots, J\}$ DO IN PARALLEL
    $w_j^n = \Phi V_j\, \underline{w}_j^n \in V_j$: $a(w_j^n, v_j) = \underbrace{\langle F, v_j\rangle - a(u^n, v_j)}_{\gamma_j \;=:\; \bar{\gamma}_j} \quad \forall v_j \in V_j$

    $\Psi_j = \Phi V_j$ (a basis of V_j)

    $\underline{w}_j^n \in \mathbb{R}^{N_j}$: $a_j(\tilde{w}_j^n, v_j) = a(u - u^n, v_j)$

    $(V_j^T K V_j)\, \underline{w}_j^n = V_j^T (\underline{f} - K\underline{u}^n) = V_j^T \underline{d}^n$

    $C_j\, \underline{\tilde{w}}_j^n = V_j^T \underline{d}^n$

  END FOR

$$u^{n+1} = u^n + \tau \sum_{j=1}^{J} \tilde{w}_j^n = u^n + \tau \underbrace{\sum_{j=1}^{J} \tilde{P}_j}_{=\,\tilde{P}} (u - u^n)$$

$$\underline{u}^{n+1} = \underline{u}^n + \tau \sum_{j=1}^{J} V_j\, \underline{\tilde{w}}_j^n$$

END FOR

Exact ASM is a special case of the inexact ASM:

$a_j(\cdot,\cdot) := a(\cdot,\cdot)$

$\tilde{P}_j = P_j$

$C_j = V_j^T K V_j$