Algorithm 2.3: Iterative Substructuring via preconditioned Richardson iteration $\tau C_c(\underline{u}_c^{n+1} - \underline{u}_c^n) + S_c \underline{u}_c^n = \underline{g}_c$, but in practise PCG or P GMRES, ...

1. FOR ALL $s = 1, p$ DO IN PARALLEL
   - generate $K_{C_s}, K_{I_s}, K_{C_s I_s}, K_{I_s C_s}, \underline{f}_{C_s}, \underline{f}_{I_s}$;
   - factorize $K_{I_s} = L_{I_s} D_{I_s} L_{I_s}^T$ or $K_{I_s} = L_{I_s} U_{I_s}$;
   - determine
   $$\underline{g}_{C_s} = \underline{f}_{C_s} - K_{C_s I_s} K_{I_s}^{-1} \underline{f}_{I_s}$$

   END FOR
2. Choose initial guess $\underline{u}_C^0$; $\underline{u}_{C_s}^0 = A_{C_s} \underline{u}_C^0$, $s = \overline{1,p}$;
3. FOR $n = 0$ STEP 1 TO CONVERGENCE DO
   - FOR ALL $s = 1, p$ DO IN PARALLEL
     - compute $\underline{v}_{C_s}^n = K_{C_s} \underline{u}_{C_s}^n - K_{C_s I_s} K_{I_s}^{-1} K_{I_s C_s} \underline{u}_{C_s}^n$;
     - compute the defect $\underline{d}_{C_s}^n = \underline{g}_{C_s} - \underline{v}_{C_s}^n$;
   - Assemble the defects
   $$\underline{d}_C^n = \sum_{s=1}^{p} A_{C_s}^T \underline{d}_{C_s}^n \quad \text{(NN communication)}$$
   - Solve the Schur-Complement preconditioning system
   $$C_C \underline{w}_C^n = \underline{d}_C^n \quad \text{(NN communication)}$$
   - FOR ALL $s = 1, p$ DO IN PARALLEL
   $$\underline{w}_{C_s}^n = A_{C_s} \underline{w}_C^n; \quad \underline{u}_{C_s}^{n+1} = \underline{u}_{C_s}^n + \tau_n \underline{w}_{C_s}^n;$$

   END FOR
4. FOR ALL $s = 1, p$ DO IN PARALLEL
   $$\underline{u}_{I_s} = K_{I_s}^{-1} (\underline{f}_{I_s} - K_{I_s C_s} \underline{u}_{C_s}^{n_*})$$
   Last iterate

   END FOR

$$K = \sum_{s=1}^{p} A_s^T K_s A_s, \quad \underline{f} = \sum_{s=1}^{p} A_s^T \underline{f}_s$$