Algorithm 2.1: Variational Formulation AltSM

0. Given $u^0 \in \mathbb{V} = \overset{\circ}{H}{}^1(\Omega) = \overset{\circ}{W}{}^1_2(\Omega)$

Iteration: $n = 0, 1, \ldots$ until CONVERGECE

1. $u^{n+1/2} = u^n + w^{n+1/2}$ with $w^{n+1/2} \in \mathbb{V}_1 = \overset{\circ}{H}{}^1(\Omega_1) \subset \overset{\circ}{H}{}^1(\Omega)$:

$$a(w^{n+1/2}, v) = \langle F, v\rangle - a(u^n, v) = a(u - u^n, v) \quad \forall v \in \mathbb{V}_1$$

$$\int_\Omega \nabla w^{n+1/2} \cdot \nabla v \, dx \qquad \int_\Omega f v \, dx$$

$$\int_{\Omega_1} \nabla w^{n+1/2} \cdot \nabla v \, dx \qquad \int_{\Omega_1} f v \, dx$$

$$\boxed{J(u^n + w^{n+1/2}) = \min_{v \in \mathbb{V}_1} J(u^n + v)}$$

$$w^{n+1/2} = P_1 \underbrace{(u - u^n)}_{=: z^n} = P_1 z^n$$

= orthogonal projection of the error $z^n = u - u^n$ to the subspace $\mathbb{V}_1 \subset \mathbb{V}$

2. $u^{n+1} = u^{n+1/2} + w^{n+1}$ with $w^{n+1} \in \mathbb{V}_2 = \overset{\circ}{H}{}^1(\Omega_2) \subset \overset{\circ}{H}{}^1(\Omega)$

$$a(w^{n+1}, v) = \langle F, v\rangle - a(u^{n+1/2}, v) = a(u - u^{n+1/2}, v) \quad \forall v \in \mathbb{V}_2$$

$$\int_{\Omega_2} \nabla w^{n+1} \cdot \nabla v \, dx \qquad \int_{\Omega_2} f v \, dx$$

$$\boxed{J(u^{n+1/2} + w^{n+1}) = \min_{v \in \mathbb{V}_2} J(u^{n+1/2} + v)}$$

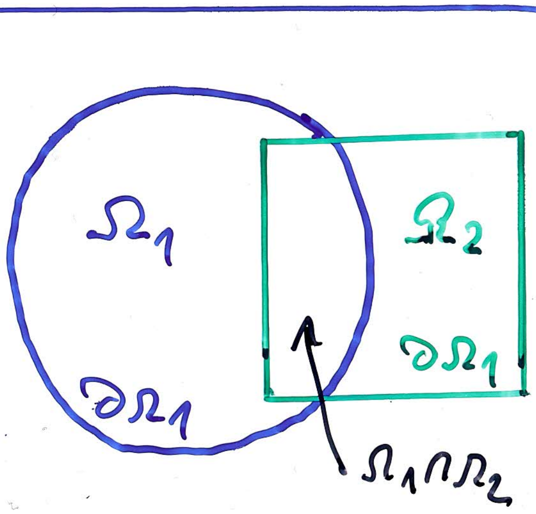
$$w^{n+1} = P_2 (u - u^{n+1/2}) = P_2 z^{n+1/2}$$

$$a(u, v) = \int_\Omega \nabla u \cdot \nabla v \, dx$$

$$\langle F, v\rangle = \int_\Omega f v \, dx$$

$P_i : \mathbb{V} \to \mathbb{V}_i$ – Ritz projection:

$$a(P_i u, v) = a(u, v) \quad \forall v \in \mathbb{V}_i$$



Reference:

Sobolev S.L.: Schwartz algorithm in the theory of elasticity. Dokl. Akad. Nauk S.S.S.R., 1936, 4, 235–238 (in Russian)