Beziehung zur

Finiten Differenzen Methode (FDM)

1D Modellproblem, $h_k = h$, $\underline{f}_h$ ... approximierter Lastvektor (Trapezregel)

FE-System:

$$K_h \, \underline{u}_h \;=\; \underline{f}_h$$

$$\begin{aligned}
\frac{1}{h}\Big[2\,u_1 - u_2\Big] &= h\,f(x_1) + \frac{g_0}{h} \\
\frac{1}{h}\Big[-u_{i-1} + 2u_i - u_{i+1}\Big] &= h\,f(x_i) && \text{für } 1 < i < n_h \\
\frac{1}{h}\Big[-u_{n_h-1} + u_{n_h}\Big] &= \frac{h}{2}\,f(x_{n_h}) + g_1
\end{aligned}$$

$$\begin{aligned}
-\frac{1}{h^2}\Big[g_0 - 2\,u_1 + u_2\Big] &= f(x_1) \\
-\frac{1}{h^2}\Big[u_{i-1} - 2u_i + u_{i+1}\Big] &= f(x_i) && \text{für } 1 < i < n_h \\
-\frac{2}{h}\Big[g_1 - \frac{u_{n_h} - u_{n_h-1}}{2\,h}\Big] &= f(x_{n_h})
\end{aligned}$$

Obiges Schema entsteht aus der Gleichung

$$-u''(x) \;=\; f(x)$$

mittels

- **zentralem Differenzenquotienten**

$$u''(x_i) \;\approx\; \frac{1}{h^2}\Big[u(x_{i-1}) - 2u(x_i) + u(x_{i+1})\Big] \quad \text{für } i < n_h$$

- **zwei einseitigen Differenzenquotienten (geschachtelt)**

$$u''(x_{n_h}) \;\approx\; \frac{1}{h}\Big[\underbrace{g_1}_{=u'(x_{n_h})} - \underbrace{\frac{u(x_{n_h}) - u(x_{n_h-1})}{h}}_{\approx u'(x_{n_h}-1)}\Big]$$