Die Voraussetzungen (V0)–(V2) sind für unser Bsp. (7) aus Abs. 2.2 erfüllt. Tatsächlich,

(V0) $|\langle F, v\rangle| = \left|\int_a^b f(x)v(x)\,dx + \alpha_b g_b v(b)\right| \leq$

$\leq \|f\|_0 \|v\|_0 + |\alpha_b|\,|g_b|\,|v(b)|$

NR: $v(b) = \int_a^b v'(x)\,dx = \int_a^b v'(x)\cdot 1\,dx$ $\quad (v(a)=0)$

$|v(b)| = \left|\int_a^b v'\cdot 1\,dx\right| \leq \sqrt{\int_a^b |v'|^2 dx}\,\sqrt{\int_a^b 1^2 dx}$

$\leq \sqrt{b-a}\;\|v\|_1$

$\|v\|_0 \leq \|v\|_1$

$\leq \|f\|_0 \|v\|_1 + |\alpha_b|\,|g_b|\sqrt{b-a}\,\|v\|_1 = c\,\|v\|_1$

mit $c = \|f\|_0 + |\alpha_b|\,|g_b|\sqrt{b-a}$.

(V1) $a(v,v) = \int_a^b v'\cdot v'\,dx + \underbrace{\alpha_b}_{\geq 0} v^2(b) \geq \int_a^b (v'(x))^2 dx =$

$= \frac{1}{2}\int_a^b (v'(x))^2 dx + \frac{1}{2}\int_a^b (v'(x))^2 dx$

$\geq \frac{1}{2}\frac{1}{c_F^2}\int_a^b (v(x))^2 dx + \frac{1}{2}\int_a^b (v'(x))^2 dx$

Abs. 2.1
Friedrichs-Ungl.: $\int_a^b v^2 dx \leq c_F^2 \int_a^b (v')^2 dx \;\; \forall v \in V_0$, $\mu_1 = \min\left\{\frac{1}{2c_F^2}, \frac{1}{2}\right\}$

(V2) $|a(w,v)| = \left|\int_a^b w'v'\,dx + \alpha_b w(b)v(b)\right| \leq$

$\leq \left|\int_a^b w'(x)v'(x)\,dx\right| + |\alpha_b|\,|w(b)|\,|v(b)| \leq$

$\wedge$ Cauchy

$\leq \sqrt{\int_a^b |w'(x)|^2 dx}\,\sqrt{\int_a^b |v'(x)|^2 dx} + |\alpha_b|\,|w(b)|\,|v(b)|$

NR

$\leq \|w\|_1 \cdot \|v\|_1 + \alpha_b (b-a)\,\|w\|_1 \|v\|_1$

$= \underbrace{(1 + \alpha_b(b-a))}_{=:\,\mu_2} \|w\|_1 \|v\|_1 \quad \forall w, v \in V_0.$