Solution of the BE system $A_h \underline{v}_h = \underline{f}_h$!

$\Longrightarrow$ PCG-method with the preconditioner

$$C_h = A_{1h} \qquad \text{(we will omit } h\text{)}$$

Initialitation:

$\underline{v}_0 \in \mathbb{R}^n$ - given initial guess;

$\underline{d}_0 := \underline{f} - A\underline{v}_0$

$\underline{w}_0 := C^{-1}\underline{d}_0 = A_1^{-1}\underline{d}_0 = n^{-1} F \Lambda^{-1} F^H \underline{d}_0;$

$\underline{s}_0 := \underline{w}_0$

Iteration: $k = 0, 1, \ldots, k_{max} = I(\varepsilon)$:

$(\underline{w}_k, \underline{d}_k) \le \varepsilon^2 (\underline{w}_0, \underline{d}_0) \Longrightarrow$ STOP;

$\alpha_{k+1} = (\underline{w}_k, \underline{d}_k) / (A\underline{s}_k, \underline{s}_k);$

$\underline{v}_{k+1} = \underline{v}_k + \alpha_{k+1}\underline{s}_k;$

$\underline{d}_{k+1} = \underline{d}_k - \alpha_{k+1} A \underline{s}_k;$

$\underline{w}_{k+1} = C^{-1}\underline{d}_{k+1} = n^{-1} F \Lambda^{-1} F^H \underline{d}_{k+1};$

$\beta_{k+1} = (\underline{w}_{k+1}, \underline{d}_{k+1}) / (\underline{w}_k, \underline{d}_k);$

$\underline{s}_{k+1} = \underline{w}_{k+1} + \beta_{k+1}\underline{s}_k$

$\Longrightarrow$ **Results:**

1. $\|\underline{v} - \underline{v}_k\|_A \le q_k \|\underline{v} - \underline{v}_0\|_A$,

   with

   $$q_k = \frac{2q^k}{1+q^{2k}}, \quad q = \frac{1-\sqrt{\xi}}{1+\sqrt{\xi}}, \quad \xi = \frac{1}{\operatorname{cond}_2(A_1^{-1}A)},$$

   $$\operatorname{cond}_2(A_1^{-1}A) \le \overline{\gamma}/\underline{\gamma} = O(1) \quad \text{(L.6.4)}$$

2. $I(\varepsilon) = \#\,\text{Iterations} = O(\ln \varepsilon^{-1})$,

   $Q(\varepsilon) = \#\,\text{Operations} = O(h^{-2}\ln\varepsilon^{-1})$,

   $M = \text{Memory} = O(h^{-2})$.