Example 5.3: Hypersingular operator D given on a cirale Ω with the radius r, i.e.

$$\Gamma = \partial\Omega := \{ x = x(t) := r\,(\cos 2\pi t, \sin 2\pi t)^T \in \mathbb{R}^2 : t \in [0,1) \}.$$

$$Du(y) := -\int_\Gamma \frac{\partial E}{\partial n_y \partial n_x}(x,y)\,(u(x) - u(y))\,ds_x$$

$$\overset{(mms)}{=} -\frac{1}{2\pi r^2} \int_\Gamma \frac{|x-y|^2 x^T y - 2\,x^T(x-y)\,y^T(x-y)}{|x-y|^4}\,(u(x)-u(y))\,ds_x$$

$x = x(t)$
$y = y(t)$

$$\overset{(mm6)}{=} -\frac{1}{4r} \int_0^1 \frac{u(t) - u(s)}{\sin^2 \pi (t-s)}\,dt.$$

Therefore, the EVP

$$D u_k = \lambda_k u_k, \quad k \in \mathbb{Z}$$

has the following solution

- eigenfunctions: $u_k = e^{i 2\pi k t}$
- eigenvalues: $\lambda_k = \begin{cases} 0 & \text{for } k = 0, \\ \frac{1}{2r}|k| & \text{for } k \neq 0. \end{cases}$

Remark 5.4:

D: Eigenf. corr. to small EV are of low frequency!
EF corr. to large EV are of high frequency!

V: EF corr. to small EV ($\searrow 0$) are of high frequency!
EF corr. to Large EV are of Low frequency!