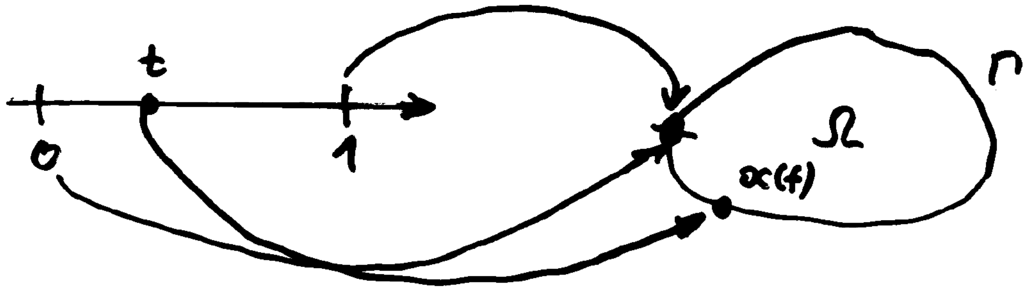
- <u>The 2D case:</u>

For $d=2$, the Sobolev–Slobodeckij spaces $H^s(\Gamma)$ can be defined by Fourier-series for 1-periodic functions. Indeed, let

$$\Gamma := \{x = x(t) \in \mathbb{R}^2 : 0 \le t < 1 \ \wedge \ |\dot{x}(t)| \ge \varkappa > 0\}$$

be a simply connected curve with the 1-periodic parameter representation $(x(0) = x(1))$:



Then, every function $u(\cdot)$ given on Γ can be interpreted as 1-periodic function on $\mathbb{R}$, i.e. $u(t) := u(x(t))$, $t \in \mathbb{R}$; $u(t) = u(t+1)$, $u(0) = u(1)$.
For a fixed $s \in \mathbb{R}$, we define

$$H^s(\Gamma) := \overline{C^\infty_{1\text{-per.}}(\underset{x}{\Gamma})}^{\|\cdot\|_s} = \overline{C^\infty_{1\text{-per}}(\underset{t}{\mathbb{R}})}^{\|\cdot\|_s},$$

where

$$\|u\|_s := \Big(|\hat{u}_0|^2 + \sum_{\substack{\kappa \in \mathbb{Z} \\ \kappa \neq 0}} |\hat{u}_\kappa|^2 |\kappa|^{2s} \Big)^{1/2}$$

with

$$u(t) = u(x(t)) = \sum_{\substack{\kappa \in \mathbb{Z} \\ \kappa \neq 0}} \hat{u}_\kappa \, e^{i 2\pi t} \quad (i^2 = -1),$$

$$\hat{u}_\kappa = \int_0^1 u(t)\, e^{-i 2\pi t}\, dt \text{ – Fourier coefficient,}$$

$\kappa \in \mathbb{Z}$