- **Computation of the matrix entries $\tilde{b}_{ij}$!**

$$\tilde{b}_{ij} = \int_{\Gamma_j} \frac{\partial E}{\partial n_x}(x,y)\, dS_x \Big|_{y=\vec{y}_i \in \mathbb{R}^3} = \int_{\Gamma_j} \frac{\partial E}{\partial n_x}(x, y_i)\, dS_x$$

$$E(x,y) = \frac{1}{4\pi} \frac{1}{|x-y|} = \frac{1}{4\pi} \left(\sum_{k=1}^{3} (x_k - y_k)^2 \right)^{-1/2}$$

$x = (x_1, x_2, x_3)$
$y = (y_1, y_2, y_3)$ notation !

$$\frac{\partial E}{\partial x_i}(x,y) = \frac{1}{4\pi}\left(-\frac{1}{2}\right)\left(\sum_{k=1}^{3}(x_k - y_k)^2\right)^{-3/2} 2(x_i - y_i)$$

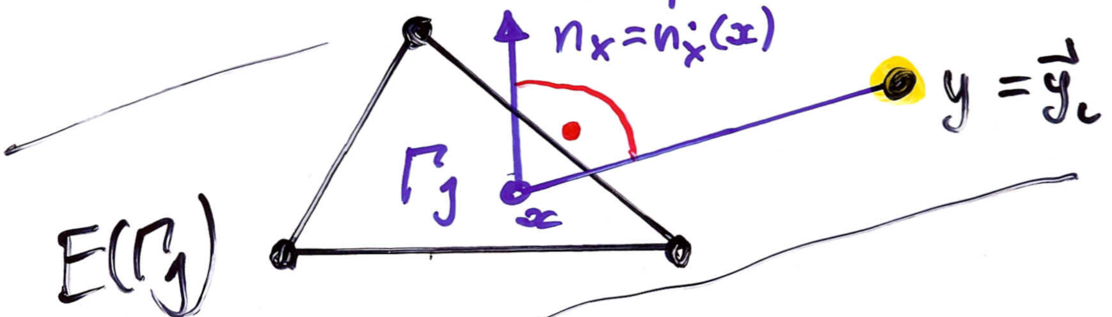
$$= -\frac{1}{4\pi} \frac{(x_i - y_i)}{|x-y|^3}$$

$$\frac{\partial E}{\partial n_x}(x,y) = (\nabla_x E(x,y), n_x) = -\frac{1}{4\pi} \frac{(x-y, n_x(x))}{|x-y|^3}$$

$$= -\frac{1}{4\pi} \int_{\Gamma_j} \frac{(x-y, n_x)}{|x-y|^3} dS_x \Big|_{y=\vec{y}_i = (y_{1i}, y_{2i}, y_{3i})}$$

To compute this integral, we study the following cases:

1. y is located in the $E(\Gamma_j)$ plane:



$n_x \perp x - y \implies (x-y, n_x) = 0$

In particular, we have $\boxed{\tilde{b}_{ii} = 0}$