Computation of the matrix entries a_{ij}:

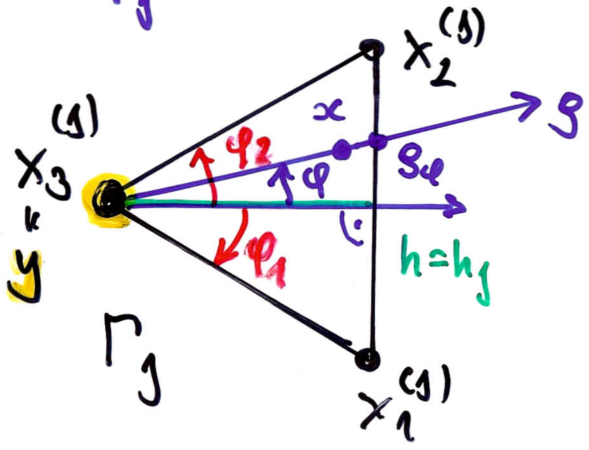
$$a_{ij} = \int_{\Gamma_j} E(x, y_i)\, ds_x = \frac{1}{4\pi} \int_{\Gamma_j} \frac{ds_x}{|x - y_i|}$$

$$\overset{y=y_i}{=} \frac{1}{4\pi} \int_{\Gamma_j} \frac{ds_x}{|x-y|} = \dots \quad \text{for } y \in \mathbb{R}^3 \,!$$

To compute this integral, we study the following cases:

1. y coincide with a vertex of the triangel Γ_j:

$$\int_{\Gamma_j} E(x,y)\, ds_x = \frac{1}{4\pi} \int_{\Gamma_j} \frac{ds_x}{|x-y|}$$



$$= \frac{1}{4\pi} \int_{\varphi_1}^{\varphi_2} \int_0^{\frac{h}{\cos\varphi}} \frac{1}{\varrho} \underbrace{\varrho\, d\varrho\, d\varphi}_{ds_x}$$

$$= \frac{1}{4\pi} \int_{\varphi_1}^{\varphi_2} \frac{h}{\cos\varphi}\, d\varphi$$

$\cos\varphi = h / \varrho_\varphi$

$\varrho_\varphi = \dfrac{h}{\cos\varphi}$

$$= \frac{h}{4\pi} \frac{1}{2} \ln \frac{1+\sin\varphi}{1-\sin\varphi} \Bigg|_{\varphi_1}^{\varphi_2}$$

AC: $\left(\ln \dfrac{1+\sin\varphi}{1-\sin\varphi}\right)' = \dots = \dfrac{2}{\cos\varphi}$

$$\boxed{\int_{\Gamma_j} E(x,y)\, ds_x = \frac{h_j}{8\pi} \ln \frac{1+\sin\varphi}{1-\sin\varphi} \Bigg|_{\varphi_1}^{\varphi_2}}$$