• **Problems** from a classical point of view:

$$\frac{\partial^2}{\partial x^2}E(x,y) \quad \nexists \quad \text{for } x \to y \; !$$

Let $\varepsilon > 0$ arbitrary and let us consider

$$\int_0^1 \frac{\partial^2}{\partial x^2}E(x,y)\,u(x)\,dx = \int_0^{y-\varepsilon} + \int_{y-\varepsilon}^{y+\varepsilon} + \int_{y+\varepsilon}^{1} =$$

$\frac{\partial^2 E}{\partial x^2} = 0$ in $(0, y-\varepsilon)$ $\qquad$ $\frac{\partial^2 E}{\partial x^2} = 0$ in $(y+\varepsilon, 0)$

2× integration by parts

$$= \int_{y-\varepsilon}^{y+\varepsilon} u(x)\frac{\partial^2}{\partial x^2}E(x,y)\,dx =$$

$$= u\frac{\partial E}{\partial x}\Big|_{y-\varepsilon}^{y+\varepsilon} - u'E\Big|_{y-\varepsilon}^{y+\varepsilon} + \int_{y-\varepsilon}^{y+\varepsilon} \underbrace{u''(x)}_{\substack{\| \\ -f(x)}}E(x,y)\,dx$$

$$= u\frac{\partial E}{\partial x}\Big|_{y-\varepsilon}^{y+\varepsilon} - u'E\Big|_{y-\varepsilon}^{y+\varepsilon} - \underbrace{\int_{y-\varepsilon}^{y+\varepsilon} f(x)E(x,y)\,dx}$$

$\exists\, \xi_\varepsilon \in (y-\varepsilon, y+\varepsilon) \to \|$ mean value ther.

$$2\varepsilon f(\xi_\varepsilon)E(\xi_\varepsilon, y)$$

$$= \left(-\frac{1}{2}u(y+\varepsilon) - \frac{1}{2}u(y-\varepsilon) - \frac{1-\varepsilon}{2}\left(u'(y+\varepsilon) - u'(y-\varepsilon)\right) - \right.$$
$$- 2\varepsilon f(\xi_\varepsilon)E(\xi_\varepsilon, y)\Big)$$

$$\xrightarrow{\varepsilon \to 0} -u(y) \qquad (f \in C^1,\; u \in C^2)$$

i.e. we have shown that

$$-\int_0^1 \frac{\partial^2}{\partial x^2}E(x,y)u(x)\,dx = -u(y), \text{ i.e.,}$$

(4) $\quad -\underbrace{\frac{\partial^2}{\partial x^2}E(x,y)}_{\text{FS}} = \underbrace{\delta(x-y)}_{\text{Dirac's delta function}} \quad$ in $\mathcal{D}'$