Basics: Network topologies & Introduction to MPI

1. We are given 8 vertices (nodes, processors).

   (a) Find a graph with diameter 3 and maximal vertex degree 3, i. e., find a network topology with at most 3 edges (links) per node where the longest path includes 3 edges.

   (b) Do graphs exist that have diameter 2 and maximal vertex degree 3? If so, specify one.

2. Consider 2^n given vertices. Find a graph that has diameter n and maximal vertex degree n.

   Hint: Hypercube

3. Modify the program from the Lecture that computes $\ln 2$ such that the rounding errors are minimized by summing in reverse direction.

4. Write a parallel code which approximates π using the partial sum

$$s_n = \sum_{k=1}^{n} (-1)^{k+1} \frac{1}{2k-1}$$

   for $n = 10^6$ and the identity $\frac{\pi}{4} = \lim_{n\to\infty} s_n$.

5. Consider the linear system

$$\begin{bmatrix} 2+2\alpha_1 & -1+\alpha_1 & 0 & 0 & 0 \\ -1+\alpha_1 & 2+2\alpha_2+2\alpha_1 & -1+\alpha_2 & 0 & 0 \\ 0 & -1+\alpha_2 & 2+2\alpha_2 & -1 & 0 \\ 0 & 0 & -1 & 2+2\alpha_3 & -1+\alpha_3 \\ 0 & 0 & 0 & -1+\alpha_3 & 2+2\alpha_3 \end{bmatrix} \underline{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

   with a stochastic matrix. The parameters α_1, α_2, and α_3 are independent from each other and equally distributed in [0, 1].

   (a) Show that the system matrix is positive definite

   (b) Determine the mean value, the maximum, and the minimum of the components of the solution vector $\underline{x}$ empirically. Perform a *parallel* stochastic simulation with 10^6 runs using up to 8 processors.

   (c) Determine the speed-up and the efficiency on all processors, where the load is equally distributed.

   (d) Same as (b), where the processors have different computing power (3 with 500 MHz, 5 with 800 MHz). How should the load be distributed?

6. Write a code to determine the Euclidean inner product $\underline{a} \cdot \underline{b}$ of two vectors $\underline{a}$ and $\underline{b}$ *in parallel.*