T U T O R I A L

"Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

Tutorial 12 Friday, Jun 20, 2008 (Time : $8^{\underline{30}}$ – $9^{\underline{15}}$ Room : SR T 1010)

5 Linear Elasticity

5.1 The Basic Equations

41 Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with Lipschitz continuous boundary $\Gamma := \partial\Omega$, and let $f \in [L_2(\Omega)]^3$, and $t \in [L_2(\Gamma)]^3$. We define the right handside F of an elastic BVP (see the lectures, Chapter 3, Box (2)) by

$$\langle F\,,\,v\rangle := \int_\Omega f^T v\,\mathrm{d}x + \int_\Gamma t^T v\,\mathrm{d}s \quad \forall v \in V := \left[H^1(\Omega)\right]^3 .$$

Show, that F is in V^*, i. e., that F is linear and bounded.

42 Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with Lipschitz continuous boundary $\Gamma := \partial\Omega$, and let the displacement $v = (v_1, v_2)^T \in [H^1(\Omega)]^2$. Let the strain $\varepsilon(v)$ be defined as $\varepsilon(v) = \frac{1}{2}\left(\nabla v + \nabla v^T\right)$. Calculate the set R for which there holds $v \in R \Leftrightarrow \varepsilon(v) = 0$.

43* Consider the iteration (see lectures, Theorem 3.7, Equation (11))

$$(u_{n+1}\,,\,v)_1 = (u_n\,,\,v)_1 + \tau\left(\langle F\,,\,v\rangle - a(u_n, v)\right) \quad \forall v \in V_0\,,$$

(Richardson with Laplace Preconditioner), where (see lectures, Chapter 3, Box (2))

$$\begin{aligned} (u\,,\,v)_1 &= \int_\Omega \nabla u^T \nabla v\,\mathrm{d}x\,, \\ a(u,v) &= \int_\Omega \varepsilon(u)^T D \varepsilon(v)\,\mathrm{d}x\,, \\ \langle F\,,\,v\rangle &= \int_\Omega f^T v\,\mathrm{d}x + \int_\Gamma t^T v\,\mathrm{d}s\,. \end{aligned}$$

Outline the scheme of how to apply a Finite Element discretization to this iteration. Which systems have to be solved, and which matrix-times-vector multiplications occur within the FE-discretized iteration?