T U T O R I A L

"Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

Tutorial 07 Friday, May 16, 2008 (Time : $8^{\underline{30}} – 9^{\underline{15}}$ Room : SR T 1010)

25* Consider the mixed variational problem: Find $(u, \lambda) \in X \times \Lambda$ such that

$$\begin{aligned} a(u, v) + b(v, \lambda) &= \langle F , v\rangle \quad \forall v \in X , \\ b(u, \mu) &= \langle G , \mu\rangle \quad \forall \mu \in \Lambda , \end{aligned}$$

where $F \in X^*$ and $G \in \Lambda^*$ are given. Let $A : X \to X^*$ and $B : X \to \Lambda^*$ be the related operators to $a(\cdot, \cdot)$ and $b(\cdot, \cdot)$, and let the assumptions of Theorem 2.4 (*Brezzi*) be satisfied. Show that the bilinearform

$$l(\xi, \eta) := \langle L\,\xi , \eta\rangle$$

with

$$L := \begin{pmatrix} A & B^* \\ B & 0 \end{pmatrix}, \ \xi := \begin{pmatrix} u \\ \lambda \end{pmatrix}, \ \eta := \begin{pmatrix} v \\ \mu \end{pmatrix}, \text{ and } \|\xi\|_{X\times\Lambda} = \left(\|u\|_X^2 + \|\lambda\|_\Lambda^2\right)^{1/2}$$

satisfies the assumptions of Theorem 1.5 (*Babuska-Aziz*), if $a(\cdot, \cdot)$ is elliptic on the whole space X, i. e., if there exists $\alpha_1 > 0$ such that $a(v, v) \geq \alpha_1 \|v\|_X^2$ for all $v \in X$.

Hint: The LBB-condition

$$\exists \mu_1 > 0\ \forall \xi = (u, \lambda) \in X \times \Lambda : \ \sup_\eta \frac{l(\xi, \eta)}{\|\eta\|} \geq \mu_1 \|\xi\|$$

can be shown by choosing $\eta = (v, \mu)$ such that $\mu = -2\lambda$, and $v = u + w$ where $w \in X$ is the solution of the adjoint problem $a(y, w) = b(y, \lambda)$ for all $y \in X$.

26 Let the assumptions of Theorem 2.7 be fulfilled. Additionally we assume $\operatorname{Ker} B_h \subset \operatorname{Ker} B$, and define $Z_h(G) = \{v_h \in X_h \mid b(v_h, \mu_h) = \langle G , \mu_h\rangle \ \ \forall \mu_h \in \Lambda_h\}$. Show that there holds the estimate

$$\|u - u_h\|_X \leq \left(1 + \frac{\alpha_2}{\tilde{\alpha}_1}\right) \inf_{v_h \in Z_h(G)} \|u - v_h\|_X . \tag{4.56}$$

27 Let the operators $B^* : \Lambda \to X^*$ and $B_h^* : \Lambda_h \to X_h^*$ be defined by

$$\langle B^*\mu , v\rangle = b(v, \mu) \quad \forall v \in X\ \forall \mu \in \Lambda$$

and

$$\langle B_h^*\mu_h , v_h\rangle = b(v_h, \mu_h) \quad \forall v_h \in X_h\ \forall \mu_h \in \Lambda_h .$$

Show that, if there exists a linear operator $\Pi_h : X \to X_h$ with

$$b(\Pi_h v - v, \mu_h) = 0 \quad \forall v \in X\ \forall \mu_h \in \Lambda_h ,$$

then there holds $\operatorname{Ker} B_h^* \subset \operatorname{Ker} B^*$.