T U T O R I A L

"Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

Tutorial 06 Friday, May 2, 2008 (Time : $8^{\underline{30}}$ – $9^{\underline{15}}$ Room : SR T 1010)

4.2 Mixed Finite Element Methods

22 Show directly (without using Theorem 2.4 (*Brezzi*)), that under the assumptions of Theorem 2.4 (*Brezzi*) the homogeneous mixed variational problem

$$\begin{aligned} a\,(u, v) + b\,(v, \lambda) &= 0 \quad \forall\, v \in X \\ b\,(u, \mu) &= 0 \quad \forall\, \mu \in \Lambda \end{aligned}$$

has only the trivial solution $(u, \lambda) = (0, 0) \in X \times \Lambda$!

23 Consider the problem: Find $u \in U$ such that for given $F \in V^*$ there holds

$$a(u, v) = \langle F\,,\, v\rangle \quad \forall v \in V\,.$$

Let the assumptions of Theorem 1.5 (*Babuska-Aziz*) be satisfied, and let $U_h \subset U$ and $V_h \subset V$ be finite dimensional subspaces. Further, we assume

$$\exists \tilde{\mu}_1 > 0 : \quad \inf_{\substack{u_h \in U_h \\ u_h \neq 0}} \sup_{\substack{v_h \in V_h \\ v_h \neq 0}} \frac{a(u_h, v_h)}{\|u_h\|_U \|v_h\|_V} \geq \tilde{\mu}_1\,, \tag{4.47}$$

and

$$\forall v_h \in V_h,\; v \neq 0\; \exists u_h \in U_h : \quad a(u_h, v_h) \neq 0\,. \tag{4.48}$$

Show, that there exists a unique solution to the variational problem:

$$\text{Find } u_h \in U_h : \quad a(u_h, v_h) = \langle F\,,\, v_h\rangle \quad \forall v_h \in V_h\,, \tag{4.49}$$

Also show, that the discretization error can be estimated from above by

$$\|u - u_h\|_U \leq \left(1 + \frac{\mu_2}{\tilde{\mu}_1}\right) \inf_{w_h \in U_h} \|u - w_h\|_U\,. \tag{4.50}$$

24* Let X and Λ be real Hilbert spaces and $B : X \to \Lambda^*$ a bounded linear operator. Show, that B satisfies the LBB-condition

$$\exists \beta_1 > 0 : \inf_{\substack{v \in \Lambda \\ v \neq 0}} \sup_{\substack{\tau \in X \\ \tau \neq 0}} \frac{\langle B\tau\,,\, v\rangle}{\|\tau\|\,\|v\|} \geq \beta_1\,,$$

if and only if there exists $c = \text{const} > 0$ such that for all $v^* \in \Lambda^*$ there exists a $\tau \in X$ such that $B\tau = v^*$ and $\|\tau\|_X \leq c\,\|v^*\|_{\Lambda^*}$.