T U T O R I A L

"Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

Tutorial 04 Friday, April 18, 2008 (Time : $8^{\underline{30}}$ – $9^{\underline{15}}$ Room : SR T 1010)

3 Nonlinear Variational Problems and Variational Inequalities

14 Let us consider the abstract nonlinear variational problem (15) from Transparency 05 under the assumption made there. Show that there exists a unique solution $u \in V_0$ of the nonlinear variational problem (15) and that the fixed point iteration (17) converges to this solution.

15 Let us consider the abstract nonlinear variational problem (15) from Transparency 05 under the assumption made there, and its finite element approximation: find $u_h \in V_{0h} \subset V_0$ such that

$$a(u_h, v_h) = \langle f, v_h \rangle \quad \forall v_h \in V_{0h}. \tag{3.29}$$

Show the Cea-like discretization error estimate

$$\|u - u_h\|_{V_0} \leq \frac{\mu_2}{\mu_1} \inf_{w_h \in V_{0h}} \|u - w_h\|_{V_0}, \tag{3.30}$$

where the μ_1 and μ_2 are the monotonicity and the Lipschitz constants, respectively.

16 Show that the variational inequality (19) is equivalent to the minimization problem (20) if the bilinear form is additionally symmetric (see Transparency 05) !

17* Show that if g additionally is continuous on $\bar{\Omega}$, a solution $u \in U \cap C^2(\bar{\Omega})$ of the obstacle problem (MP) $\equiv$ (VI) satisfies the PDE (in)equilities $-\Delta u \geq f$, $u \geq g$, $(\Delta u + f)(u - g) = 0$ in Ω and $u = 0$ on Γ (see Transparency 06) !