TUTORIAL

"Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

Tutorial 02 Friday, April 4, 2008 (Time : $8^{30} - 9^{55}$ Room : SR / T 1010)

1.3 Scalar Elliptic BVP of the Fourth Order.

04 Show that the first biharmonic BVP

$$u \in V_0 := H_0^2(\Omega) : \int_{\Omega} \Delta u(x) \Delta v(x) dx = \int_{\Omega} f(x) v(x) dx \ \forall v \in V_0$$
(1.7)

has a unique solution, where $H_0^2(\Omega) := \{ v \in H^2(\Omega) : u = \frac{\partial u}{\partial n} = 0 \text{ on } \Gamma \}$! <u>Hint:</u> Use the Lax-Milgram theorem and the following inequility

$$\int_{\Omega} |\Delta v(x)|^2 dx \ge \mu_1 ||v||^2_{H^2(\Omega)}$$
(1.8)

that is valid for all $v \in H_0^2(\Omega)$.

- 05^* Show inequality (1.8) !
- 06 Give the variational formulations for BVPs of the second, the third and the fourth kind mentioned in Example 1.3 !
- 07^* Prove that the so-called Kirchhoff plate bilinear form

$$a(u,v) := \int_{\Omega} \left\{ \Delta u(x) \Delta v(x) + (1-\sigma) \left[2 \frac{\partial^2 u}{\partial x_1 \partial x_2} \frac{\partial^2 v}{\partial x_1 \partial x_2} - \frac{\partial^2 u}{\partial x_1^2} \frac{\partial^2 v}{\partial x_2^2} - \frac{\partial^2 u}{\partial x_2^2} \frac{\partial^2 v}{\partial x_1^2} \right] \right\} dx$$
(1.9)

is identical to the biharmonic bilinear form given in (1.7) in the case of the first BVP (i.e., on $H_0^2(\Omega)$), where $\sigma \in (0, 1)$ is a given material parameter (Poisson-coefficient).

- 08 Derive the natural boundary conditions for the plate bilinear form (1.9)? <u>Hint:</u> Use Schwarz' theorem, i.e. $\frac{\partial^2 u}{\partial x_1 \partial x_2} = \frac{\partial^2 u}{\partial x_2 \partial x_1}$, and two times partial integrations !
- 09 Derive a mixed variational formulation for the first biharmonic boundary value problem (1.7) by introducing the new variable $w = \Delta u$!