T U T O R I A L

"Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

Tutorial 01 Friday, March 14, 2008 (Time : $8^{\underline{30}}$ – $9^{\underline{15}}$ Room : SR / T 1010)

1 Introduction to Mixed Variational Formulations: Examples

1.1 Scalar Elliptic BVP of the Second Order.

01 Formulate the mixed variational formulation of the mixed BVP

$$-\Delta u = f \text{ in } \Omega,\ u = g \text{ on } \Gamma_1,\ \frac{\partial u}{\partial n} = 0 \text{ on } \Gamma_2$$

for the Poisson-equation with given f, g, Γ_1 and Γ_2, where $\Gamma_1 \cap \Gamma_2 = \emptyset$ and $\Gamma_1 \cup \Gamma_2 = \Gamma = \partial\Omega$!

1.2 The Stokes Equations

02 Let us consider a two-dimensional, steady state (stationary) flow of a highly viscous, incompressible fluid that can be described by the Stokes Equations

$$-\nu\Delta u + \nabla p = f \quad \text{in } \Omega, \tag{1.1}$$

$$\text{div} u = 0 \quad \text{in } \Omega. \tag{1.2}$$

Assume that the velocity u can be represented by a so-called (scalar) stream-function ψ as

$$u = \mathbf{curl}\,\psi \tag{1.3}$$

with

$$\mathbf{curl}\,\psi = \begin{pmatrix} \frac{\partial\psi}{\partial x_2} \\ -\frac{\partial\psi}{\partial x_1} \end{pmatrix}$$

Show that

$$\text{div}\, u = 0, \tag{1.4}$$

and derive the following relation from the equations (1.1):

$$\nu\Delta^2\psi = \text{curl}\, f \quad \text{in } \Omega, \tag{1.5}$$

where the so-called scalar curl is now defined by the relation

$$\operatorname{curl} f = \frac{\partial f_2}{\partial x_1} - \frac{\partial f_1}{\partial x_2}. \tag{1.6}$$

03 Which boundary conditions must be added to the biharmonic equation (1.5) such that the velocity $u = \mathbf{curl}\,\psi$ fulfils the no-slip boundary conditions $u = 0$ on the boundary $\Gamma = \partial\Omega$.
Hint: See Example 1.3 for possible boundary conditions for the biharmonic equation !