- LAX-MILGRAM's theorem gives

$\exists!\, u \in V_0 : (2) \equiv (3)$ + fixed point iteration

provided that the <u>STANDARD ASSUMPTIONS</u> are fulfilled:

1. $F \in V_0^*$ OK (mms)
2. $a(\cdot,\cdot) : V_0 \times V_0 \longrightarrow \mathbb{R}$ – bilinear form:

2a) <u>V_0-elliptic:</u> $a(v,v) \geq \mu_1 \|v\|_1^2 \;\forall v \in V_0$?

First we immediately obtain

(4) $\bullet\; a(v,v) \geq \lambda_{min}(D) \int_\Omega |\varepsilon(v)|^2 dx = \lambda_{min}(D) \underline{\int_\Omega \varepsilon_{ij}(v)\varepsilon_{ij}(v)dx} \geq$?

$(4)_{iso}$ $\bullet\; a(v,v) \geq 2\mu \underline{\int_\Omega \varepsilon_{ij}(v)\varepsilon_{ij}(v)\,dx} \geq$?

In order to estimate $\int_\Omega \varepsilon_{ij}(v)\,\varepsilon_{ij}(v)$ further, we need the so-called **KORN's inequality** ($\downarrow$) !

2b) <u>V_0-bounded:</u> $|a(u,v)| \leq \mu_2 \|u\|_1 \|v\|_1 \;\forall u,v \in V_0$

(5) $\bullet\; |a(u,v)| \leq \lambda_{max}(D) \left(\int_\Omega |\varepsilon(u)|^2 dx\right)^{1/2} \left(\int_\Omega |\varepsilon(v)|^2 dx\right)^{1/2}$

$$\leq \underbrace{\lambda_{max}(D)}_{=\mu_2} \quad \|u\|_1 \cdot \|v\|_1$$

$$\int_\Omega |\varepsilon(u)|^2 dx = \int_\Omega \varepsilon_{ij}(u)\,\varepsilon_{ij}(u)\,dx = \int_\Omega u_{i,i}^2\,dx + 2\cdot\frac{1}{4}\int_\Omega \underbrace{(u_{1,2}+u_{2,1})^2}_{\leq 2(u_{1,2}^2+u_{2,1}^2)} dx + \ldots$$

$$\leq |u|_1^2 := \sum_{i=1}^3 \sum_{j=1}^3 \int_\Omega (u_{i,j})^2 dx \leq \|u\|_1^2$$

- isotropic, homogeneous:

$(5)_{iso}$ $$|a(u,v)| \leq 3\lambda \left(\int_\Omega \varepsilon_{kk}^2(u)dx\right)^{0.5} \left(\int_\Omega \varepsilon_{kk}^2(v)dx\right)^{0.5} + 2\mu \left(\int_\Omega |\varepsilon(u)|^2 dx\right)^{0.5} \left(\int_\Omega |\varepsilon(v)|^2 dx\right)^{0.5}$$

$$\leq (3\lambda + 2\mu)\|u\|_1 \|v\|_1, \text{ i.e. } \mu_2 = 3\lambda + 2\mu.$$