# 2.2. Formulations in the Case of a S+P Bilinear Form $a(\cdot,\cdot)$

- Let the <u>Assumptions</u>

$$(26)\begin{cases} \bullet\ (3)_{1)}\ \ f\in X^*,\ g\in\Lambda^* \\ \bullet\ (3)_{2)}\ \ a(\cdot,\cdot),\ b(\cdot,\cdot) - \text{continuous}: \alpha_2, \beta_2 \\ \bullet\ (3)_{4)}\ \ a(\cdot,\cdot) - V_0\text{-elliptic}: \alpha_1 \\ \bullet\ V_g = V(g) := \{v\in X:\ b(v,\mu) = \langle g,\mu\rangle\ \forall\mu\in M\} \neq \emptyset \end{cases}$$

be fulfilled, i.e. (3) <u>without</u> LBB, but with $V_g \neq \emptyset$, and let the bilinear form $a(\cdot,\cdot)$ be Symmetric + Positive:

$$(27)\begin{cases} \bullet\ \ a(u,v) = a(v,u)\ \ \forall u,v\in X & \textbf{S} \\ \bullet\ \ a(v,v) > 0\ \ \forall v\in X:\ v\neq 0 & \textbf{P} \end{cases}$$

- We consider now the following CMP:

(28)

$$\boxed{\begin{array}{l} \text{Find } u\in V_g:\ J(u) = \inf\limits_{v\in V_g} J(v) \\ \qquad\qquad \boxed{v\in X:\ b(v,\mu) = \langle g,\mu\rangle\ \forall\mu\in\Lambda} \\ \qquad\qquad = \text{constraint in equality form} \\ \text{Find } u\in X:\ \ J(u) = \inf\limits_{v\in X} J(v) \\ \qquad\qquad \text{s.t. } b(u,\mu) = \langle g,\mu\rangle\ \forall\mu\in\Lambda \end{array}}$$

with the Ritz energy functional

$$(29)\quad J(v) = \frac{1}{2}a(v,v) - \langle f,v\rangle .$$