$$\int_\Omega \Delta^2 u \cdot v \, dx = \int_\Omega \Delta\Delta u \cdot v \, dx =$$
$$\int_\Omega \Delta u \cdot \Delta v \, dx + \int_\Gamma \partial_n \Delta u \cdot v \, ds - \int_\Gamma \Delta u \cdot \partial_n v \, ds = \int_\Omega f v \, dx$$

Possible BC: $\longrightarrow$ see step 3

- essential : $u = g_0$ on Γ
- essential : $\partial_n u = g_1$ on Γ
- natural : $\Delta u = g_2$ on Γ
- natural : $\partial_n \Delta u = g_3$ on Γ

possible combinations

Possible BVP for the biharmonic PDE $\Delta^2 u = f$:

1st BVP : $u = g_0$ and $\partial_n u = g_1$ on Γ
2nd BVP : $u = g_0$ and $\Delta u = g_2$ on Γ
3rd BVP : $\partial_n u = g_1$ and $\partial_n \Delta u = g_3$ on Γ
4th BVP : $\Delta u = g_2$ and $\partial_n \Delta u = g_3$ on Γ
mixed BVP : e.g. $u = \partial_n u = 0$ on Γ_1 and $\Delta u = \partial_n \Delta u = 0$ on Γ_2
$\Gamma = \Gamma_1 \cup \Gamma_2$

Mixed VF: $\longrightarrow$ see Tutorial 2

The idea is the same as in Example 1.2:
Introduce an additional unknown

$$\sigma = \Delta u$$

.... EXERCISE $\boxed{09}$ of TUTORIAL 2