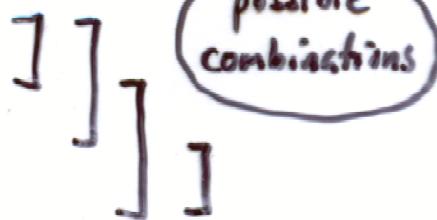


$$\int_{\Omega} \Delta^2 u \cdot v \, dx = \int_{\Omega} \Delta \Delta u \cdot v \, dx =$$

$$\int_{\Omega} \Delta u \cdot \Delta v \, dx + \int_{\Gamma} \partial_n \Delta u \cdot v \, ds - \int_{\Gamma} \Delta u \cdot \partial_n v \, ds = \int_{\Omega} f v \, dx$$

Possible BC: → see step ③

- essential:  $u = g_0$  on  $\Gamma$
- essential:  $\partial_n u = g_1$  on  $\Gamma$
- natural:  $\Delta u = g_2$  on  $\Gamma$
- natural:  $\partial_n \Delta u = g_3$  on  $\Gamma$



Possible BVP for the biharmonic PDE  $\Delta^2 u = f$ :

1st BVP:  $u = g_0$  and  $\partial_n u = g_1$  on  $\Gamma$

2nd BVP:  $u = g_0$  and  $\Delta u = g_2$  on  $\Gamma$

3rd BVP:  $\partial_n u = g_1$  and  $\partial_n \Delta u = g_3$  on  $\Gamma$

4th BVP:  $\Delta u = g_2$  and  $\partial_n \Delta u = g_3$  on  $\Gamma$

Mixed BVP: e.g.  $u = \partial_n u = 0$  on  $\Gamma_1$  and  $\Delta u = \partial_n \Delta u = 0$  on  $\Gamma_2$   
 $\Gamma = \Gamma_1 \cup \Gamma_2$

Mixed VF: → see Tutorial 2

The idea is the same as in Example 1.2:

Introduce an additional unknown

$$G = \Delta u$$

... EXERCISE [05] of TUTORIAL 2