# ÜBUNGEN ZU NUMERIK ZEITABHÄNGIGER PROBLEME

für den 26.11.2007

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31. Show that

$$p(t) = p(t_j + s\,\tau) = \sum_{i=0}^{k-1} (-1)^i \binom{-s}{i} \nabla^i f_j$$

is a polynomial of degree $\leq k-1$ with

$$p(t_i) = f_i \quad \text{for all } i = j-k+1, \ldots, j$$

and $t_i = t_j - (j-i)\,\tau$.

32. Assume that the following Lipschitz condition is satisfied

$$\|f(t,w) - f(t,v)\| \leq L\,\|w - v\| \quad \text{for all } t, v, w.$$

Show that the implicit Adams methods for the initial value problem

$$\begin{aligned} u'(t) &= f(t, u(t)) \quad \text{for } t > 0, \\ u(0) &= u_0 \end{aligned}$$

are well-defined for sufficiently small step sizes $\tau$.

33. Let $\gamma_i$, $i = 0, 1, \ldots$, be the coefficients of the Adams-Bashforth methods:

$$\gamma_i = (-1)^i \int_0^1 \binom{-s}{i}\, ds.$$

Let $G(t) = \sum_{i=0}^{\infty} \gamma_i t^i$ be the generating function of the sequence $(\gamma_i)_{i \in \mathbb{N}_0}$.
Show:

$$G(t) = -\frac{t}{(1-t)\ln(1-t)}.$$

Hint: Use the binomial theorem $(1+x)^r = \sum_{i=0}^{\infty} \binom{r}{i} x^i$ for $r \in \mathbb{R}$.

34. Assume the notations of Exercise 33. Show the recurrence relations

$$\gamma_k + \frac{1}{2}\gamma_{k-1} + \frac{1}{3}\gamma_{k-2} + \ldots \frac{1}{k+1}\gamma_0 = 1.$$

Hint: Use the identity

$$-\frac{\ln(1-t)}{t} G(t) = \frac{1}{1-t}$$

and compare the coefficients of $t^k$.

35. Let $\gamma_i^*$, $i = 0, 1, \ldots$, be the coefficients of the implicit Adams methods:

$$\gamma_i^* = (-1)^i \int_0^1 \binom{-s+1}{i} \, ds.$$

Show

$$\gamma_0^* = 1$$

and

$$\gamma_k^* + \frac{1}{2}\gamma_{k-1}^* + \frac{1}{3}\gamma_{k-2}^* + \ldots \frac{1}{k+1}\gamma_0^* = 0 \quad \text{for } k \geq 1.$$

36. Let $\kappa_i$ and $\kappa_i^*$ be the coefficients of the explicit Nyström methods and the Milne-Simpson methods, respectively, given by

$$\kappa_i = (-1)^i \int_{-1}^1 \binom{-s}{i} \, ds, \quad \kappa_i^* = (-1)^i \int_{-1}^1 \binom{-s+1}{i} \, ds.$$

Show:

$$\gamma_i^* = \gamma_i - \gamma_{i-1}, \quad \kappa_i = 2\gamma_i - \gamma_{i-1}, \quad \kappa_i^* = 2\gamma_i^* - \gamma_{i-1}^* \quad \text{for } k \geq 0$$

with $\gamma_1 = \gamma_1^* = 0$.