ÜBUNGEN ZU NUMERIK ZEITABHÄNGIGER PROBLEME

für den 19.11.2007

---

25. Construct the adjoint methods of the explicit midpoint rule and the implicit midpoint rule.

26. Let ϕ be the increment function of a one-step method. The increment function of the adjoint one-step method is denoted by ϕ^*. Show: $(\phi^*)^* = \phi$.

27. Show that a collocation method with symmetrically distributed collocation points is symmetric.

    Hint: Use the representation of the coefficients a_{ij} and b_j from Exercise 19.

28. Show that explicit Runge-Kutta methods are not symmetric.

29. Verify the Aitken-Neville algorithm for extrapolation:

$$T_{j,k+1} = T_{j,k} + \frac{T_{j,k} - T_{j-1,k}}{n_j/n_{j-k} - 1}.$$

    Hint: Show for the associated interpolation polynomials

$$T_{j,k+1}(s) = \frac{T_{j,k}(s)(s - \tau_{j-k}) - T_{j-1,k}(s)(s - \tau_j)}{\tau_j - \tau_{j-k}}.$$

30. Show that the method obtained by local extrapolation applied to a Runge-Kutta method is also a Runge-Kutta method. It suffices to consider Richardson extrapolation: $k = 2$, $n_1 = 1$, $n_2 = 2$.