ÜBUNGEN ZU NUMERIK ZEITABHÄNGIGER PROBLEME

für den 29.10.2007

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14. Construct the tableau of Butcher's 3-stage Lobatto III method.

15. Let $c_1 = 0, c_2, \ldots, c_s$ and $b_1, \ldots, b_s$ be the coefficients of the corresponding Radau quadrature formula. Show: The conditions $a_{1j} = 0$ for $j = 1, \ldots, s$ and $D(s-1)$ imply $C(s)$.

16. Let $c_1, c_2, \ldots, c_{s-1}, c_s = 1$ and $b_1, \ldots, b_s$ be the coefficients of the corresponding Radau quadrature formula. Show: The conditions $a_{is} = 0$ for $i = 1, \ldots, s$ and $C(s-1)$ imply $D(s)$ and vice versa.

17. Let $c_1 = 0, c_2, \ldots, c_{s-1}, c_s$ and $b_1, \ldots, b_s$ be the coefficients of the corresponding Radau quadrature formula. Ehle's Radau IA method is determined by the conditions $D(s)$ (instead of $C(s)$ for Butcher's Radau I method). Show for Ehle's Radau IA method: $a_{i1} = b_1$ for $i = 1, \ldots, s$.

18. Let $c_1, c_2, \ldots, c_{s-1}, c_s = 1$ and $b_1, \ldots, b_s$ be the coefficients of the corresponding Radau quadrature formula. Ehle's Radau IIA method is determined by the conditions $C(s)$ (instead of $D(s)$ for Butcher's Radau II method). Show for Ehle's Radau IIA method: $a_{sj} = b_j$ for $j = 1, \ldots, s$.

19. For solving the initial value problem

$$\begin{aligned} u'(t) &= f(t, u(t)), \quad t \in [t_0, T], \\ u(t_0) &= u_0 \end{aligned}$$

consider the following so-called collocation method: Let $s \in \mathbb{N}$ and $c_1, c_2, \ldots, c_s \in \mathbb{R}$ be distinct. The approximate solution u_1 for $u(t_0 + \tau)$ is given by

$$u_1 = p_s(t_0 + \tau),$$

where p_s is that polynomial of degree s, which satisfies the conditions

$$\begin{aligned} &p_s(t_0) = u_0 \\ &p_s'(t_0 + c_i\tau) = f(t_0 + c_i\tau, p_s(t_0 + c_i\tau)), \quad i = 1, 2, \ldots, s. \end{aligned}$$

Show: The collocation method can be represented as a Runge-Kutta method with

$$a_{ij} = \int_0^{c_i} l_j(c)\, dc, \quad b_j = \int_0^1 l_j(c)\, dc,$$

where $l_j(c)$ is the j-th Lagrange polynomial, given by

$$l_j(c) = \prod_{k \neq j} (c - c_k) \Big/ \prod_{k \neq j} (c_j - c_k).$$

Hint: Show and use $p_s'(t_0 + c\tau) = \sum_j k_j \cdot l_j(c)$ with $k_i = p_\ell'(t_0 + c_j\tau)$, $p_s(t_0 + c_i\tau) = u_0 + \tau \int_0^{c_i} p_s'(t_0 + c\tau)\ dc$, and $p_s(t_0 + \tau) = u_0 + \tau \int_0^1 p_s'(t_0 + c\tau)\ dc$.