ÜBUNGEN ZU NUMERIK ZEITABHÄNGIGER PROBLEME

für den 22.10.2007

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8. Determine all 2-stage explicit Runge-Kutta methods, described by the tableau

$$\begin{array}{c|cc} 0 & & \\ c_2 & a_{21} & \\ \hline & b_1 & b_2 \end{array}$$

   with $c_2 = a_{21}$ of (consistency) order 2.

9. Determine the principal error term of the 2-stage explicit Runge-Kutta method, described by the tableau

$$\begin{array}{c|cc} 0 & & \\ \theta & \theta & \\ \hline & 1 - \frac{1}{2\theta} & \frac{1}{2\theta} \end{array}.$$

   What value do you suggest for θ?

10. Consider an s-stage explicit Runge-Kutta method with tableau

$$\begin{array}{c|ccccc} 0 & & & & & \\ c_2 & a_{21} & & & & \\ c_3 & a_{31} & a_{32} & & & \\ \vdots & \vdots & \vdots & \ddots & & \\ c_s & a_{s,1} & a_{s,2} & \dots & a_{s,s-1} & \\ \hline & b_1 & b_2 & \dots & b_{s-1} & b_s \end{array}.$$

   and $c_i = \sum_j a_{ij}$. Show that the conditions for a method of order 3

$$\begin{aligned} \sum_j b_j &= 1 \\ 2\sum_{j,k} b_j a_{jk} &= 1 \\ 3\sum_{j,k,l} b_j a_{jk} a_{jl} &= 1 \\ 6\sum_{j,k,l} b_j a_{jk} a_{kl} &= 1 \end{aligned}$$

are equivalent to

$$\begin{aligned} \sum_j b_j &= 1 \\ 2\sum_j b_j c_j &= 1 \\ 3\sum_j b_j c_j^2 &= 1 \\ 6\sum_{j,k} b_j a_{jk} c_k &= 1 \end{aligned}$$

11. Determine all 3-stage explicit Runge-Kutta methods, described by the tableau

$$\begin{array}{c|ccc} 0 & & & \\ c_2 & a_{21} & & \\ c_3 & a_{31} & a_{32} & \\ \hline & b_1 & b_2 & b_3 \end{array}$$

with $c_2 = a_{21}$ and $c_3 = a_{31} + a_{32}$ of (consistency) order 3.

Hint: Use c_2 and c_3 as arbitrary parameters and determine the other coefficients from the second set of order conditions of exercise 10

12. Apply the classical 4-stage Runge-Kutta method, given by the tableau

$$\begin{array}{c|cccc} 0 & & & & \\ 1/2 & 1/2 & & & \\ 1/2 & 0 & 1/2 & & \\ 1 & 0 & 0 & 1 & \\ \hline & 1/6 & 1/3 & 1/3 & 1/6 \end{array},$$

to the scalar linear differential equation $u'(t) = \lambda \cdot u(t)$ for a given constant $\lambda \in \mathbb{C}$ and show that the approximate solutions u_1 at $t = t_0 + \tau$ can be written in the following form:

$$u_1 = R(\tau \cdot \lambda) \cdot u_0.$$

Determine the function $R(z)$.

13. Determine the conditions on the coefficients of an s-stage explicit Runge-Kutta method that guarantee that the principal error term is of the form $C_p \tau^{p+1}$ with C_p independent of τ, if applied to differential equations of the special form $u'(t) = f(t)$.