Chapter 3
Hyperbolic Partial Differential Equation

3.1. IBVP for Hyperbolic PDEs and their **Discretization in Space and Time**

- <u>LVF of hyperbolic PDEs:</u> $V = V_0 \subset H^1(\Omega)$, $H = L_2(\Omega)$

(1)
$$\boxed{\begin{array}{l}\text{Find } u \in L_2((0,T),V) \text{ with } u' \in L_2((0,T),H) \\ \text{and } u'' \in L_2((0,T),V^*) \text{ such that} \\ \frac{d^2}{dt^2}(u(t),v)_H + a(u(t),v) = \langle F(t),v\rangle \;\forall v \in V \;\dot{\forall} t \in (0,T) \\ \text{IC: } u(0) = u_0 \text{ and } u'(0) = v_0.\end{array}}$$

- Existence and uniqueness: see Lecture Notes
- <u>Model Problem:</u> Derive the LVF (1)!
  $\frac{\partial^2 u}{\partial t^2}(x,t) - \frac{\partial^2 u}{\partial x^2}(x,t) = f(x,t)$, $(x,t) \in Q_T = (0,1) \times (0,T)$
  BC: e.g. $u(0,t) = u(1,t) = 0$, $t \in (0,T)$
  IC: $u(x,0) = u_0(x)$, $\frac{\partial u}{\partial t}(x,0) = v_0(x)$, $x \in [0,1]$

- <u>Semidiscretization in space by the VML:</u>

$V_h = V_{0h} = \text{span}\{\varphi_i : i = \overline{1,N_h}\} \subset V = V_0$ - FE-subspace
$u_h(x,t) = \sum_{k=1}^{N_h} u_k(t)\varphi_k(x) \longleftrightarrow \underline{u}_h(t) = [u_k(t)]_{k=\overline{1,N_h}} \in [H^2(0,T)]^{N_h}$

(2)
$$\boxed{\begin{array}{l}\text{Find } \underline{u}_h(t) = [u_k(t)]_{k=\overline{1,N_h}} \in [H^2(0,T)]^{N_h}: \\ M_h \underline{u}_h''(t) + K_h \underline{u}_h(t) = \underline{f}_h(t) \;\dot{\forall} t \in (0,T) \\ \text{IC: } M_h \underline{u}_h(0) = \underline{u}_{0h},\; M_h \underline{u}_h'(0) = \underline{v}_{0h}\end{array}}$$

= 2nd order ODE system