2.3.7. Rigorous Stability Analysis for more general Problems

- So far, stability was only studied for

(37) $$u'(t) = \lambda u(t),$$

but motivated by Remark 2.17 (T20-21):

- <u>Extension to linear systems $u'(t) = Ju(t) + \cancel{f(t)}$:</u>
  - Consider

    (53) $$u'(t) = Ju(t) \quad \text{with } J \in \mathbb{R}^{n \times n}$$

    System (53) is <u>dissipative</u> iff

    (54) $$(Jv, v) \leq 0 \quad \forall v \in \mathbb{R}^n$$
  - RKF applied to (53) gives

    (55) $$u_{j+1} = R(\tau J)\, u_j$$

    RKF (55) is <u>contractive</u> iff

    (56) $$\|w_{j+1} - v_{j+1}\| = \|R(\tau J)(w_j - v_j)\| \leq \|w_j - v_j\|$$

    i.e. iff

    (56) $$\|R(\tau J)\| \leq 1$$

- <u>The case that J is normal, i.e. $J^* J = J J^*$:</u>
  - <u>Lemma 2.26:</u>

    For normal matrices J, the system (53) $u' = Ju$ is dissipative iff

    (57) $$\operatorname{Re} \lambda \leq 0 \quad \forall \lambda \in \sigma(J).$$

    <u>Proof:</u>
    - $(\cdot,\cdot)_{\mathbb{R}^n} \longmapsto (\cdot,\cdot)_{\mathbb{C}^n}$

      $\forall z = x + iy,\ z' = x' + iy' \in \mathbb{C}$:

      $(z, z') = (z, z')_{\mathbb{C}} := (x, x') + (y, y') + i[(x', y) - (x, y')] \in \mathbb{C}$