Example 2.16: Our model problem (5)

$$f(t,u) = M_h^{-1}\left[f_h(t) - K_h u_h(t)\right], \quad K_h \text{ SPD}$$

$u = u_h,\ w = w_h,\ v = v_h,\ \|\cdot\| := \|\cdot\|_{M_h}$

$$\begin{aligned}
&\left(f(t,w) - f(t,v),\ w - v\right)_{M_h} = \\
&\quad = -\left(M_h^{-1} K_h\,(w_h - v_h),\ w_h - v_h\right)_{M_h} \\
&\quad \le -\lambda_{\min}\left(M_h^{-1}K_h\right)\|w_h - v_h\|_{M_h}^2 \le 0
\end{aligned}$$

$\le \lambda_{\max}(M_h^{-1}K_h)$

Thus, while the original Lip-const. $\|M_h^{-1}K_h\|_{M_h}$ is large, the one-side Lip-const. is 0, i.e. the ODE is dissipative.

Moreover, the one-side Lip-const. $L = -\lambda_{\min}(M_h^{-1}K_h) < 0$

i.e. $\|w(t) - v(t)\|_{M_h} \le e^{-(t-t_0)\lambda_{\min}(M_h^{-1}K_h)}\,\|w(t_0) - v(t_0)\|.$