

Choice of the coefficients $\{c_i\}, \{a_{ij}\}, \{b_j\}$:

- Aim:** Choose $\{c_i\}, \{a_{ij}\}, \{b_j\}$ such that the corresponding l -stage explicit RKF has the maximal consistency order $p(l)$!

$$\max_{\{a_{ij}\}, \{b_j\}} p(l) \quad (c_l = \sum_{j=1}^{l-1} a_{lj})$$

- Result:** the symbol $|$ denotes the so-called Butcher barriers

l	1	2	3	4	5	6	7	8	9	$l \geq 10$
$p(l)$	1	2	3	4	4	5	6	6	7	$\leq l-3$

- Necessary number $l(p)$ of stages to achieve the consistency order p :

p	1	2	3	4	5	6	7	8	9	10
$l(p)$	1	2	3	4	6	7	9	≥ 11	≥ 12	≥ 13

- Guinness Book of Records:**

1975: Curtis: 18-stage explicit RKF of order 10

1978: Hairer: 17-stage explicit RKF of order 10

- Example of a 4-stage explicit RKF of order 4**
= the classical RKF = the RKF:

0				
1/2	1/2			
1/2	0	1/2		
1	0	0	1	
	1/6	1/3	1/3	1/3

The classical RKF applied to the quadrature problem $u'(t) = f(t)$

gives the SIMPSON-rule: $u_p(t+h) = u(t) + \tau \left[\frac{1}{2} f(t) + \frac{4}{3} f(t+\frac{\tau}{2}) + \frac{1}{3} f(t+\tau) \right]$