The PCG-method: $Ku = f \longmapsto C^{-1}Ku = C^{-1}f$

- By the substitutions ($\longmapsto$) C SPD

$$
\begin{aligned}
(w,v)_2 &\longmapsto (w,v)_C := (Cw,v)_2\\
K &\longmapsto C^{-1}K\\
f &\longmapsto C^{-1}f\\
d &\longmapsto C^{-1}d =: w \quad \text{(preconditioned defect)}
\end{aligned}
$$

we obtain:

<u>Initial settings:</u> $w^0 = C^{-1}(f - Ku^0) = C^{-1}d^0$

<u>Iteration:</u> For $n = 0, 1, 2, \ldots$:

$$
p^n = \begin{cases} w^0 & \text{for } n = 0,\\ w^n + \beta^{(n-1)} p^{n-1}, \text{ with } \beta^{(n-1)} = \dfrac{(w^n, w^n)_C}{(w^{n-1}, w^{n-1})_C} & \text{for } n \geq 1, \end{cases}
$$

$$
u^{n+1} = u^n + \alpha^{(n)} p^n, \text{ with } \alpha^{(n)} = \frac{(w^n, w^n)_C}{(w^{n-1}, w^{n-1})_C},
$$

$$
w^{n+1} = w^n - \alpha^{(n)} C^{-1}Kp^n \quad (d^{n+1} = d^n - \alpha^{(n)} Kp^n)
$$

- Symmetry = s.a. of $C^{-1}K$ w.r.t. $(\cdot,\cdot)_C$:

$$
(C^{-1}Kv, w)_C = (Kv, w) = (v, Kw) = (Cv, C^{-1}Kw) = (v, C^{-1}Kw)_C
$$

- <u>Energy functional:</u>

$$
\begin{aligned}
J(v) &= \tfrac{1}{2}(C^{-1}Kv, v)_C - (C^{-1}f, v)_C\\
&= \tfrac{1}{2}(CC^{-1}Kv, v) - (CC^{-1}f, v) = \tfrac{1}{2}(Kv, v) - (f, v)
\end{aligned}
$$

- Therefore, one immediately obtain the convergence result from Theorem 1.36 by replacing

$\varkappa(K)$ by $\varkappa(C^{-1}K)$!