

Example 1.34: BPX, MDS

Multilevel structure: $\tau_1 \rightarrow \dots \rightarrow \tau_{l-1} \rightarrow \tau_l \rightarrow \dots \rightarrow \tau_L : h_l = h_{l-1}/2$

initial mesh $\tau_1 : h_1$



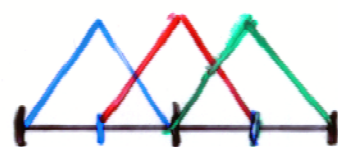
$$V_1 = \text{span } \Phi_1$$

$$\Phi_1 = [\phi_{1,i}]_{i=1, \overline{N_1}}$$



τ_2

$$h_2 = h_1/2$$



$$V_2 = \text{span } \Phi_2$$

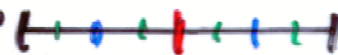
$$\Phi_2 = [\phi_{2,i}]_{i=1, \overline{N_2}}$$



$$\phi_{l-1,i} = \frac{1}{2} \phi_{l,2i-1} + \phi_{l,2i} + \frac{1}{2} \phi_{l,2i+1}$$

$$V_L = \text{span } \Phi_L$$

finest mesh $\tau_L : h_L = h_{L-1}/2$



$$\Phi_L = [\phi_{L,i}]_{i=1, \overline{N_L}}$$

Multilevel space splitting:

$$V_h = \sum_{l=1}^L \sum_{i=1}^{N_l} V_{l,i} \quad , \quad V_{l,i} = \text{span} \{ \phi_{l,i} \}$$

- exact ASM = Multilevel Diagonal Scaling (MDS)

$$a(\phi_{l,i}, \phi_{l,i}) w_{l,i} = \langle F, \phi_{l,i} \rangle - a(u_L^{(n)}, \phi_{l,i})$$



$$b_{l,i}(\phi_{l,i}, \phi_{l,i}) := 2^{2L} (\phi_{l,i}, \phi_{l,i})_0 \approx a(\phi_{l,i}, \phi_{l,i})$$

- inexact ASM : BPX = Bramble-Pasciak-Xu
- MSM : $\rightarrow$ Multigrid V-Cycle

Results:

$$1. \text{ops}(C_h^{-1} F_h) = O(N_h) = O(N_L)$$

$$2. \kappa(C_h^{-1} K_h) = O(1) \quad (\text{Proof is non-trivial!})$$