Remark 1.33: For our model problem (Example 1.2), it can be shown that (mms: cf. Subsection 1.4.1) $\exists\, c1,\, c2 =$ const > 0:

$$c_1\, h^2\, D_h \leq K_h \leq c_2\, D_h\,, \tag{54}$$

with $D_h =$ diag K_h. This implies

$$\frac{\nu_2}{\nu_1} = \mathcal{O}(h^{-2})$$

for the **Jacobi** method ($C_h = D_h$), which is no essential improvement compared to $C_h = I_h$. The same is true for the **Gauss-Seidel** method.

Optimal Preconditioners C_h:

1. $\kappa(C_h^{-1}\, K_h) = \mathcal{O}(1)$ for $h \to \infty$
2. ops$(C_h^{-1} * \underline{d}_h) = \mathcal{O}(N_h)$

can be obtained for **multilevel** splittings of the space V_h: **BPX, MDS, MGM, AMG**, . . .
See Example 1.34 and the literature.