Fast Solvers: Tutorial — Summer Term 2007

Incomplete Factorization — **April 30**

M-matrices, Existence of ILU Preconditioners

Exercise 12

Prove that every symmetric M-matrix A is positive definite.

Exercise 13

Prove the following statement (Theorem 2.1.2 in the Lecture Notes):

Let A be an $N \times N$ M-matrix and let C be any matrix obtained from A by setting certain off-diagonal entries of A to zero. Then C is an M-matrix.

Exercise 14

Prove the following statement (Theorem 2.1.3 in the Lecture Notes):

If $A = (a_{i,j})$ is an $N \times N$ M-matrix, then there exists for every pattern $P \subseteq P_N$ a lower triangular matrix $L = (l_{i,j})$, with unit diagonal ($l_{i,i} = 1$) and an upper triangular matrix $U = (u_{i,j})$ with

$$\begin{aligned} l_{i,j} &= 0 \quad \text{if} \quad (i,j) \in P, \\ u_{i,j} &= 0 \quad \text{if} \quad (i,j) \in P, \\ r_{i,j} &= 0 \quad \text{if} \quad (i,j) \notin P, \end{aligned}$$

such that the splitting $A = LU - R$ is regular.