Beispiel 7.15:

1. $\underline{n=0}$: $\int_0^1 f(x)\,dx \approx w_0 f(x_0) = 1\cdot f\left(\frac{1}{2}\right)$ MR=Gauß1

Nichtlineares GS (6):

$$\boxed{\begin{aligned} 1 &= w_0 \\ \tfrac{1}{2} &= w_0 x_0 \end{aligned}} \quad \begin{aligned} &\Longrightarrow w_0 = 1 \\ &\Longrightarrow x_0 = \tfrac{1}{2} \end{aligned}$$

2. $\underline{n=1}$: $\int_0^1 f(x)\,dx \approx w_0 f(x_0) + w_1 f(x_1)$

Nichtlineares GS (6):

$$\boxed{\begin{aligned} 1 &= w_0 + w_1 \\ \tfrac{1}{2} &= w_0 x_0 + w_1 x_1 \\ \tfrac{1}{3} &= w_0 x_0^2 + w_1 x_1^2 \\ \tfrac{1}{4} &= w_0 x_0^3 + w_1 x_1^3 \end{aligned}} \quad \overset{\text{(MMS)}}{\Longrightarrow} \quad \begin{aligned} &w_0 = \\ &w_1 = 1 - w_0 = \\ &x_0 = \\ &x_1 = \end{aligned}$$

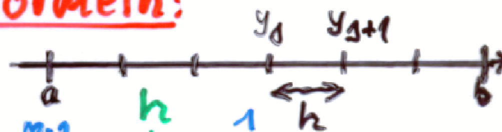
Fehler:

$$\left| \int_0^1 f(x)\,dx - \sum_{i=0}^{n} w_i f(x_i) \right| \le \frac{((n+1)!)^4 \, (1-0)^{2n+3}}{((2n+2)!)^3 (2n+3)} \max_{x\in[0,1]} \left| f^{(2n+2)}(x) \right|$$

$\underline{\text{Beweis:}}$ Hämmerlin + Hoffmann (Kap. 7, § 3)

Zusammengesetzte Gauß-Formeln:

$y_j = y_0 + jh$, $y_0 = a$, $h = \frac{b-a}{m}$



$$I[f] := \int_a^b f(x)\,dx = \sum_{j=0}^{m-1} \int_{y_j}^{y_{j+1}} f(x)\,dx = \sum_{j=0}^{m-1} \underbrace{(y_{j+1} - y_j)}_{h} \int_0^1 f(y_j + \xi h)\,d\xi$$

$$\approx \sum_{j=0}^{m-1} h \sum_{i=0}^{n} w_i f(y_j + x_i h) =: I_h[f]$$

$$|I[f] - I_h[f]| \le \underbrace{\sum_{j=0}^{m-1} h}_{= b-a} \frac{((n+1)!)^4}{((2n+2)!)^3 (2n+3)} h^{2n+2} \max_{j=0,\dots,m-1} \max |f^{(2n+2)}(y)| \xrightarrow{h\to 0} 0$$

$y \in [y_j, y_{j+1}]$