

7.3. Numerische Integration

$$\int_a^b f(x) dx \approx \sum_{i=0}^n w_i f(x_i)$$

$a \leq x_0 < x_1 < \dots < x_n \leq b$ Gewichte Stützstellen
 $\uparrow \quad \uparrow \quad \uparrow$
 $0 \leq \xi_0 < \xi_1 < \dots < \xi_n \leq 1$
 $[a, b] \leftrightarrow [0, 1] \text{ oder } [-1, 1]$

$$\int_0^1 f(a + \xi(b-a)) d\xi (b-a) \approx (b-a) \sum_{i=0}^n \tilde{w}_i \tilde{f}(\xi_i)$$

$\underbrace{\int_0^1 f(a + \xi(b-a)) d\xi}_{=: \tilde{f}(\xi)} (b-a) \approx (b-a) \sum_{i=0}^n \tilde{w}_i \tilde{f}(\xi_i)$

Zusammengesetzte (summierte) Formeln:

$$\int_a^b f(x) dx = \sum_{j=0}^{m-1} \int_{y_j}^{y_{j+1}} f(x) dx = \sum_{j=0}^{m-1} (y_{j+1} - y_j) \int_0^1 f(y_j + \xi(y_{j+1} - y_j)) d\xi$$

$a = y_0 < y_1 < \dots < y_m = b$ $[y_j, y_{j+1}] \leftrightarrow [0, 1] \text{ oder } [-1, 1]$

$$\approx \sum_{j=0}^{m-1} (y_{j+1} - y_j) \sum_{i=0}^n \tilde{w}_i f(y_j + \xi_i(y_{j+1} - y_j))$$

Mehrdimensionale numerische Integration: Ü3

1. $\Omega = [a_1, b_1] \times \dots \times [a_d, b_d] \subset \mathbb{R}^d$

1D num. Integr. $\downarrow$

$$\int_{\Omega} f(x) dx = \int_{a_1}^{b_1} \dots \left(\int_{a_d}^{b_d} f(x_1, \dots, x_d) dx_d \right) \dots dx_1 \approx \dots$$

2. $\overline{\Omega} = \bigcup_{r \in \mathbb{R}_h} \overline{\delta}_r$ mit $\overline{\delta}_r = \square \leftrightarrow \overline{\Delta} = [0, 1]^d$ ($d=2$)

$\overline{\delta}_r = \nabla \leftrightarrow \overline{\Delta} = \begin{matrix} \uparrow \xi_2 \\ \text{triangle} \\ \text{0} \quad 1 \quad \xi_1 \end{matrix}$ ($d=2$)

$x = x_{\delta_r}(\xi)$

$$\int_{\Omega} f(x) dx = \sum_{r \in \mathbb{R}_h} \int_{\overline{\delta}_r} f(x) dx = \sum_{r \in \mathbb{R}_h} \int_{\overline{\Delta}} f(x_{\delta_r}(\xi)) |\overline{\delta}_r| d\xi \approx \dots$$