2. $n=1$: $x_0 = x-h$, $x_1 = x$: Analog

$$f'(x) \approx p_1'(x) = \frac{f(x)-f(x-h)}{h} = f'(x) + O(h)$$

$$=: f_{\bar{x}}(x) = \text{rückwärtiger Differenzenquotient}$$

3. $n=2$: $x_0 = x-h$, $x_1 = x$, $x_2 = x+h$

$$p_2(y) = f(x-h)\,L_0(y) + f(x)\,L_1(y) + f(x+h)\,L_2(y)$$

$$f'(x) \approx p_2'(x) = f(x-h)\underbrace{L_0'(x)}_{=-\frac{1}{2h}} + f(x)\underbrace{L_1'(x)}_{=0} + f(x+h)\underbrace{L_2'(x)}_{=\frac{1}{2h}}$$

$$= \frac{f(x+h)-f(x-h)}{2h} \overset{\text{Taylor}}{=} f'(x) + O(h^2)$$

$$= f_{\mathring{x}}(x) = \text{zentraler Differenzenquotient}$$

Extrapolation:

Btr. z.B. zentralen Differenzenquotien in x als Fkt. von h:

$$T(h) = f_{\mathring{x}}(x) := \frac{f(x+h)-f(x-h)}{2h} \overset{\text{Taylor}}{=} f'(x) + a_1 h^2 + a_2 h^4 + \ldots$$

$$\overset{z=h^2}{=} f'(x) + a_1 z + a_2 z^2 + \ldots \quad \text{mit } z = h^2$$

$0 < z_0 = h_0^2 < z_1 = h_1^2 < \ldots < z_n = h_n^2$ Stützstellen

$T_0 = T(h_0)$ $T_1 = T(h_1)$ $T_n = T(h_n)$ Werte

$\Rightarrow$ Interpolationspolynom: $p_n(z) = \sum_{i=0}^{n} T_i L_i(z)$

$\Rightarrow$ $T(h) = \tilde{T}(z) \approx p_n(z)$

$\Rightarrow$ <u>Extrapolation:</u> $h = 0 \notin [h_0, h_n]$

$$f'(x) = T(0) \approx p_n(0) = f'(x) + O(h_n^{2(n+1)})$$