Der Neville-Algorithmus zur Berechnung von $p(x)$:

- Sei $p_{i_0,i_1,\dots,i_k}(x) \in \mathcal{P}_k$ das Interpolationspolynom vom Grade $\le k$ mit $p_{i_0,i_1,\dots,i_k}(x_{i_j}) = f_{i_j}$, $j=0,1,\dots,k$

- **Satz 7.2:**

$$p_i(x) = f_i\,,\quad i=\overline{0,n}$$

$$p_{i_0,i_1,\dots,i_k}(x) = \frac{(x-x_{i_0})\,p_{i_1,\dots,i_k}(x) - (x-x_{i_k})\,p_{i_0,i_1,\dots,i_{k-1}}(x)}{x_{i_k}-x_{i_0}}$$

Beweis: induktiv

$$p_{i_0,i_1,\dots,i_k}(x_{i_0}) = p_{i_0,i_1,\dots,i_{k-1}}(x_{i_0}) = f_{i_0}$$

$$p_{i_0,i_1,\dots,i_k}(x_{i_k}) = p_{i_1,\dots,i_k}(x_{i_k}) = f_{i_k}$$

Für $j=1,2,\dots,k-1$ gilt:

$$p_{i_0,i_1,\dots,i_k}(x_{i_j}) = \frac{(x_{i_j}-x_{i_0})\,p_{i_1,\dots,i_k}(x_{i_j}) - (x_{i_j}-x_{i_k})\,p_{i_0,\dots,i_{k-1}}(x_{i_j})}{x_{i_k}-x_{i_0}}$$

$$= \frac{(x_{i_k}-x_{i_0})\,f_{i_j} - (x_{i_0}-x_{i_0})\,f_{i_j}}{x_{i_k}-x_{i_0}} = f_{i_j}.$$

q.e.d.

- **Neville-Algorithmus:** $p(x) = p_{0,1,\dots,n}(x)$, z.B. $n=4$

$$\boxed{\text{ops}(p(x)) = O(n^2)}$$

$$\begin{array}{lllll}
f_4 = p_4(x) & & & & \\
 & p_{34}(x) & & & \\
f_3 = p_3(x) & & p_{234}(x) & & \\
 & p_{23}(x) & & p_{1234}(x) & \\
f_2 = p_2(x) & & p_{123}(x) & & p_{01234}(x) = p(x) \\
 & p_{12}(x) & & p_{0123}(x) & \\
f_1 = p_1(x) & & p_{012}(x) & & \\
 & p_{01}(x) & & & \\
f_0 = p_0(x) & & & &
\end{array}$$

$x - x_3$

$\div\; x_4 - x_3$

$x - x_4$

Bsp.: $n=1$

f_1 $\quad x - x_1$

$\div\; x_1 - x_0$ $\quad p(x) = f_1\,\dfrac{x-x_1}{x_1-x_0} + f_0\,\dfrac{x-x_0}{x_1-x_0}$

f_0 $\quad x - x_0$