Rayleigh-Quotient:

A SPD $\quad$ EW: $\quad 0 < \lambda_1 = \lambda_{min} \leq \lambda_2 \leq \dots \leq \lambda_n = \lambda_{max}$

$\sigma(A) \subset \mathbb{R}^+ \quad$ EV: $\quad q_1, \quad q_2, \dots, q_n \in \mathbb{R}^n$

$\quad$ "

$\quad$ vONS $\quad (q_i, q_j)_2 = \delta_{ij} \quad \forall i,j = \overline{1,n}$

$$x = \sum_{i=1}^{n} (x, q_i)\, q_i \quad , \quad \|x\|_2^2 = \sum_{i=1}^{n} (x, q_i)^2$$

$$Ax = \sum_{i=1}^{n} (x, q_i)\, \lambda_i q_i \quad , \quad (Ax, x)_2 = \sum_{i=1}^{n} \lambda_i (x, q_i)^2$$

$$A = Q\, D\, Q^T \quad , \qquad Q = [q_1 q_2, \dots, q_n]$$
$$D = \operatorname{diag}[\lambda_1, \lambda_2, \dots, \lambda_n]$$

$$\lambda_1 = \lambda_{min} \leq \underbrace{\frac{(Ax, x)}{(x, x)}}_{\text{Rayleigh quotient}} = \frac{\sum_{i=1}^{n} \lambda_i (x, q_i)^2}{\sum_{i=1}^{n} (x, q_i)^2} \leq \lambda_n = \lambda_{max}$$

$$\lambda_1 = \lambda_{min}(A) = \min_{x \in \mathbb{R}^n \setminus \{0\}} \frac{(Ax, x)}{(x, x)}$$

$$\lambda_n = \lambda_{max}(A) = \max_{x \in \mathbb{R}^n \setminus \{0\}} \frac{(Ax, x)}{(x, x)}$$

$$\lambda_i = \min_{\substack{x \in \mathbb{R}^n \setminus \{0\} \\ x \perp \operatorname{span}\{q_1, \dots, q_{i-1}\}}} \frac{(Ax, x)}{(x, x)} = \max_{\substack{x \in \operatorname{span}\{q_i, \dots, q_n\} \\ x \neq 0}} \frac{(Ax, x)}{(x, x)}$$

$$R(x) := \frac{(Ax, x)}{(x, x)} : \mathbb{R}^n \setminus \{0\} \to \mathbb{R} \, ,$$

$$\nabla R(x) :=$$