Das PCG-Verfahren: C SPD, $(\cdot,\cdot) := (\cdot,\cdot)$ […]

Algorithmus 4.5: PCG = CG für $C = I$

Startschritt: = Gradientenschritt $(k=0)$

$x^{(0)} \in \mathbb{R}^n$ – geg. Startnäherung, z. B. $x^{(0)} = C^{-1} * b$;

$r^{(0)} := b - A x^{(0)}$;

$w^{(0)} := C^{-1} * r^{(0)}$;

$s^{(0)} := w^{(0)}$

Iteration:

FOR $k=0$ STEP 1 UNTIL "Convergence" DO

$$\alpha^{(k)} := \frac{(r^{(k)}, s^{(k)})}{(As^{(k)}, s^{(k)})} \underset{1)}{\overset{(mm6)}{=}} \frac{(r^{(k)}, w^{(k)})}{(As^{(k)}, s^{(k)})}$$

$$x^{(k+1)} := x^{(k)} + \alpha^{(k)} s^{(k)};$$

$$r^{(k+1)} := r^{(k)} - \alpha^{(k)} A s^{(k)};$$

$$w^{(k+1)} := C^{-1} * r^{(k+1)};$$

$$\beta^{(k)} := -\frac{(Aw^{(k+1)}, s^{(k)})}{(As^{(k)}, s^{(k)})} \underset{2)}{\overset{(mm6)}{=}} \frac{(w^{(k+1)}, r^{(k+1)})}{(w^{(k)}, r^{(k)})}$$

$$s^{(k+1)} := w^{(k+1)} + \beta^{(k)} s^{(k)} \quad \overset{A}{\perp} s^{(k)}$$

ENDFOR

"Convergence" := "$k_{max} = I(\varepsilon)$ ODER $\|x - x^{(k)}\|^2_{AC^{-1}A} \le \varepsilon^2 \|x - x^{(0)}\|^2_{AC^{-1}A}$"

$$\underbrace{\|x - x^{(k)}\|^2_{AC^{-1}A}}_{(w^{(k)}, r^{(k)})} \le \varepsilon^2 (w^{(0)}, r^{(0)})$$

1) $$\frac{(r^{(k)}, s^{(k)})}{(As^{(k)}, s^{(k)})} = \frac{(r^{(k)}, w^{(k)} + \beta^{(k)} s^{(k)})}{(As^{(k)}, s^{(k)})} \underset{\uparrow}{=} \frac{(r^{(k)}, w^{(k)})}{(As^{(k)}, s^{(k)})}$$

$$(r^{(k)}, s^{(k-1)}) = (r^{(k-1)} - \alpha^{(k-1)} A s^{(k-1)}) = (r^{(k-1)}, s^{(k-1)}) - (r^{(k-1)}, s^{(k-1)}) = \ldots$$ […]

2) $$\beta^{(k)} = -\frac{(As^{(k)}, w^{(k+1)})}{(As^{(k)}, s^{(k)})} = -\frac{(As^{(k)}, w^{(k+1)})}{(As^{(k)}, w^{(k)})} = -\frac{(r^{(k+1)} - r^{(k)}, w^{(k+1)})}{(r^{(k+1)} - r^{(k)}, w^{(k)})} = \frac{(r^{(k+1)}, w \ldots)}{(r^{(k)}, w \ldots)}$$ […]