- <u>Beweis:</u>

$$Ax = b \iff C^{-1}Ax = C^{-1}b$$

$$C^{-1/2}AC^{-1/2}y = C^{-1/2}b$$

$$(7) \iff (9)$$

ODER $M = I - \tau C^{-1}A$

$$\|M\|_C^2 := \sup_x \frac{\|Mx\|_C^2}{\|x\|_C^2} = \sup_x \frac{(CMx, Mx)}{(Cx, x)}$$

$(\cdot,\cdot) = (\cdot,\cdot)_2$

$$= \sup_x \frac{(C^{1/2}MC^{-1/2}C^{1/2}x, C^{1/2}MC^{-1/2}C^{1/2}x)}{(\underbrace{C^{+1/2}x}_{=y}, C^{1/2}x)}$$

$$= \sup_y \frac{(C^{1/2}MC^{-1/2}y, C^{1/2}MC^{-1/2}y)}{(y, y)}$$

$$= \sup_y \frac{\|C^{1/2}MC^{-1/2}y\|_2^2}{\|y\|_2^2} = \|C^{1/2}MC^{-1/2}\|_2^2.$$

Die Matrix $C^{1/2}MC^{-1/2} = I - \tau C^{-1/2}AC^{-1/2}$ ist offenbar symmetrisch. Daher gilt:

$$\|C^{1/2}MC^{-1/2}\|_2 = \varrho(C^{1/2}MC^{-1/2}) = \varrho(M) =$$

$$= \max\{|1 - \tau\lambda_{min}(C^{-1/2}AC^{-1/2})|, |1 - \tau\lambda_{max}(C^{-1/2}AC^{-1/2})|\}$$

$$\|M\|_{(S)} = \sup_x \frac{\|Mx\|_{(S)}}{\|x\|_{(S)}} \overset{!}{=} \varrho(M)\ ?$$

(sym.)

q.e.d